

CONTENTS

Part 1. Examples

1. INTRODUCTION

1.1	Definitions	3
1.2	The Gauss integral theorem	4
1.3	Vector fields	5
1.4	The Green formulas	6
1.5	The Maxwell equations	7
1.6	The equations of gas dynamics	8
1.7	The heat equation	10

2. THE WAVE EQUATION

2.1	The wave equation in R_1	11
2.2	Domain of dependence; domain of determinateness	13
2.3	The initial-boundary problem	14
2.4	The wave equation in R_3	15
2.5	Finding the solution	17
2.6	The domains $\bar{\alpha}$, $\bar{\beta}$ for the wave equation in R_3	19
2.7	The wave equation in R_2	20
2.8	The domains $\bar{\alpha}$, $\bar{\beta}$ for the wave equation in R_2	21
2.9	Dependence of the wave equation on dimension	21
2.10	Continuable initial conditions; determinism in nature	23
2.11	Wave forms	24
2.12	An initial-boundary value problem in R_3	25

3. THE POTENTIAL EQUATION

3.1	The initial-value problem for the potential equation	28
3.2	Singularity functions	29
3.3	The fundamental solution	30
3.4	Green's function of the first kind	31
3.5	Poisson's formula	32
3.6	The existence of the Green's function in R_2	35
3.7	The mean-value and the maximum-minimum properties	36

3.8	Harnack's inequality	39
3.9	H. Weyl's lemma in the simplest case	40
3.10	Mean value properties for $\Delta_n u = f$ and $\Delta_3 u + k^2 u = f$	41

4. THE HEAT EQUATION

4.1	The existence theorem for the initial-value problem	43
4.2	The uniqueness theorem for the initial-value problem	47
4.3	Counterexamples	49
4.4	Remarks	51
4.5	Initial-boundary value problems	52

Part 2. Classification into Types, Theory of Characteristics, and Normal Form

1. DIFFERENTIAL EQUATIONS OF THE FIRST ORDER

1.1	Classification into types	59
1.2	Invariance properties of \dot{C}	61
1.3	Characteristic directions	62
1.4	Normal form in the hyperbolic case for $n = 2$	63
1.5	Normal form in the elliptic case for $n = 2$	64
1.6	Normal form in the parabolic case for $n = 2$	67
1.7	Differential equations of mixed type for $n = 2$	68

2. SYSTEMS OF DIFFERENTIAL EQUATIONS OF THE FIRST ORDER

2.1	Hyperbolic systems for two independent variables	69
2.2	Characteristic manifolds; normal form	71
2.3	Normal form for the quasi-linear case	72
2.4	Theory of characteristics for general systems	75
2.5	Classification into types for simple systems	77
2.6	Normal form for elliptic systems	77

3. ON THE NECESSITY OF CLASSIFICATION INTO TYPES

3.1	The existence theorem of Cauchy—S. Kowalewski	81
3.2	The example of O. Perron	83

Part 3. Questions of Uniqueness

1. ELLIPTIC AND ELLIPTIC-PARABOLIC TYPE

1.1	The maximum-minimum principle	89
1.2	The energy-integral method	93

1.3	Treatment of existence problems by means of the maximum-minimum principle	94
1.4	<i>A priori</i> estimates	95
1.5	The analyticity of the harmonic functions	96
2. PARABOLIC TYPE		
2.1	The maximum-minimum principle	99
2.2	Counterexample	100
3. HYPERBOLIC TYPE		
3.1	The energy-integral method for the wave equation	102
3.2	The energy-integral method for general systems	103
3.3	The radiation problem	107
3.4	Proof of F. Rellich's first lemma	110
3.5	The radiation problem for the whole space	113
4. MIXED TYPE		
4.1	The energy-integral method for equations of elliptic-parabolic-hyperbolic type	115
4.2	The maximum-minimum principle for equations of elliptic-parabolic-hyperbolic type	117
4.3	Remarks	120

Part 4. Questions of Existence

1. EQUATIONS OF HYPERBOLIC TYPE IN TWO INDEPENDENT VARIABLES

1.1	The initial-value problem for linear systems in two unknown functions	125
1.2	Supplements	130
1.3	The characteristic initial-value problem	133
1.4	The initial-value problem for quasi-linear systems	136
1.5	Proof of the lemma	139
1.6	Hyperbolic systems in the form of conservation theorems	145
1.7	Riemann's method	146
1.8	An example	149

2. BOUNDARY AND INITIAL-VALUE PROBLEMS FOR EQUATIONS OF HYPERBOLIC AND PARABOLIC TYPE IN TWO INDEPENDENT VARIABLES

2.1	Posing of the problem	151
2.2	The calculus of the Laplace transform	152
2.3	Solution of the transformed problem II	154

2.4	Justification of the calculus	157
2.5	Auxiliary considerations	163
2.6	The formal calculus of Laplace transforms	168

3. EQUATIONS OF ELLIPTIC TYPE

3.1	Estimates for potentials	172
3.2	A solution of $\Delta_n u = f(x)$	173
3.3	Formulation of the general boundary-value problem	176
3.4	Outline of proof and notations	177
3.5	Existence of a W -solution	179
3.6	Differentiability of the W -solution	182
3.7	Continuous assumption of the boundary values	183
3.8	Tools	190

4. WEYL'S LEMMA FOR EQUATIONS OF ELLIPTIC TYPE

4.1	Singular integrals	195
4.2	Weyl's lemma	199

Part 5. Simple Tools from Functional Analysis Applied to Questions of Existence

1. AUXILIARY TOOLS

1.1	Banach space	209
1.2	Hilbert space	210
1.3	Bounded linear functionals in Hilbert space	214

2. SCHAUDER'S TECHNIQUE OF PROOF FOR EXISTENCE PROBLEMS IN ELLIPTIC DIFFERENTIAL EQUATIONS

2.1	Posing the problem	218
2.2	Outline of proof	219

3. THE REGULAR EIGENVALUE PROBLEM

3.1	Posing the problem	222
3.2	Equivalent formulation of the problem	222
3.3	Complete continuity of the operator	226
3.4	The expansion theorem	229

4. ELLIPTIC SYSTEMS OF DIFFERENTIAL EQUATIONS

4.1	Posing the problem	232
4.2	The Green's function of the second kind	232

4.3	Hilbert's lemma	235
4.4	Equivalent formulations of the problem	237
4.5	The homogeneous first boundary-value problem	240
4.6	The inhomogeneous first boundary-value problem	242
4.7	The general boundary-value problem with characteristic zero	244
4.8	The general boundary-value problem with arbitrary integer characteristic	246
	SOLUTIONS	251
	BIBLIOGRAPHY	256
	INDEX	257