

# Contents

## Part I: Functional analysis

Chapter I. Distribution theory . . . . .	1
1.0. Introduction . . . . .	1
1.1. Weak derivatives . . . . .	1
1.2. Test functions. . . . .	2
1.3. Definitions and basic properties of distributions . . . . .	4
1.4. Differentiation of distributions and multiplication by functions . . . . .	7
1.5. Distributions with compact support . . . . .	10
1.6. Convolution of distributions . . . . .	13
1.7. Fourier transforms of distributions . . . . .	17
1.8. Distributions on a manifold . . . . .	25
Chapter II. Some special spaces of distributions . . . . .	33
2.0. Introduction . . . . .	33
2.1. Temperate weight functions. . . . .	34
2.2. The spaces $\mathcal{B}_{p,k}$ . . . . .	36
2.3. The spaces $\mathcal{B}_{p,k}^{loc}$ . . . . .	42
2.4. The spaces $\mathcal{H}_{(s)}$ . . . . .	45
2.5. The spaces $\mathcal{H}_{(m,s)}$ . . . . .	51
2.6. The spaces $\mathcal{H}_{(s)}^{loc}(\Omega)$ when $\Omega$ is a manifold . . . . .	56

## Part II: Differential operators with constant coefficients

Chapter III. Existence and approximation of solutions of differential equations . . . . .	63
3.0. Introduction . . . . .	63
3.1. Existence of fundamental solutions . . . . .	64
3.2. The equation $P(D)u = f$ when $f \in \mathcal{E}'$ . . . . .	69
3.3. Comparison of differential operators . . . . .	71
3.4. Approximation of solutions of homogeneous differential equations . . . . .	76
3.5. The equation $P(D)u = f$ when $f$ is in a local space $\subset \mathcal{D}'_p$ . . . . .	80
3.6. The equation $P(D)u = f$ when $f \in \mathcal{D}'$ . . . . .	83
3.7. The geometric meaning of $P$ -convexity and strong $P$ -convexity . . . . .	89
3.8. Systems of differential operators . . . . .	94
Chapter IV. Interior regularity of solutions of differential equations . . . . .	96
4.0. Introduction . . . . .	96
4.1. Hypoelliptic operators . . . . .	97
4.2. Partially hypoelliptic operators . . . . .	104
4.3. Partial hypoellipticity at the boundary . . . . .	107
4.4. Estimates for derivatives of high order . . . . .	108
Chapter V. The Cauchy problem (constant coefficients) . . . . .	114
5.0. Introduction . . . . .	114
5.1. The classical existence theory for analytic data . . . . .	116
5.2. The non-uniqueness of the characteristic Cauchy problem . . . . .	120

5.3. Holmgrens' uniqueness theorem . . . . .	123
5.4. The necessity of hyperbolicity for the existence of solutions to the non-characteristic Cauchy problem. . . . .	130
5.5. Algebraic properties of hyperbolic polynomials . . . . .	132
5.6. The Cauchy problem for a hyperbolic equation . . . . .	137
5.7. A global uniqueness theorem . . . . .	142
5.8. The characteristic Cauchy problem. . . . .	151
<b>Part III: Differential operators with variable coefficients</b>	
Chapter VI. Differential equations which have no solutions . . . . .	156
6.0. Introduction . . . . .	156
6.1. Conditions for non-existence . . . . .	156
6.2. Some properties of the range . . . . .	166
Chapter VII. Differential operators of constant strength . . . . .	170
7.0. Introduction . . . . .	170
7.1. Definitions and basic properties . . . . .	170
7.2. Existence theorems when the coefficients are merely continuous . . . . .	172
7.3. Existence theorems when the coefficients are in $C^\infty$ . . . . .	173
7.4. Hypoellipticity . . . . .	176
7.5. The analyticity of the solutions of elliptic equations . . . . .	177
Chapter VIII. Differential operators with simple characteristics . . . . .	180
8.0. Introduction . . . . .	180
8.1. Necessary conditions for the main estimates . . . . .	181
8.2. Differential quadratic forms . . . . .	187
8.3. Estimates for elliptic operators . . . . .	190
8.4. Estimates for operators with real coefficients . . . . .	193
8.5. Estimates for principally normal operators . . . . .	199
8.6. Pseudo-convexity . . . . .	202
8.7. Estimates, existence and approximation theorems in $\mathcal{H}_{(s)}$ . . . . .	207
8.8. The unique continuation of singularities . . . . .	216
8.9. The uniqueness of the Cauchy problem. . . . .	224
Chapter IX. The Cauchy problem (variable coefficients) . . . . .	230
9.0. Introduction . . . . .	230
9.1. Preliminary lemmas . . . . .	230
9.2. The basic $L_2$ estimate . . . . .	234
9.3. Existence theory for the Cauchy problem. . . . .	237
Chapter X. Elliptic boundary problems . . . . .	242
10.0. Introduction . . . . .	242
10.1. Definition of elliptic boundary problems . . . . .	243
10.2. Preliminaries concerning ordinary differential operators . . . . .	246
10.3. Construction of a parametrix . . . . .	248
10.4. Local theory of elliptic boundary problems . . . . .	254
10.5. Elliptic boundary problems in a compact manifold with boundary . . . . .	258
10.6. Various extensions and remarks . . . . .	267
Appendix. Some algebraic lemmas . . . . .	275
Bibliography . . . . .	280
Index . . . . .	286
Index of notations . . . . .	287