Contents

Historical Preface *page* xi
General Outline xvi

Part I: Volume Preserving Homeomorphisms of the Cube 1

1 Introduction to Parts I and II (Compact Manifolds) 3
1.1 Dynamics on Compact Manifolds 3
1.2 Automorphisms of a Measure Space 3
1.3 Main Results for Compact Manifolds 4

2 Measure Preserving Homeomorphisms 7
2.1 The Spaces $\mathcal{M}, \mathcal{H}, \mathcal{G}$ 7
2.2 Extending a Finite Map 9

3 Discrete Approximations 13
3.1 Introduction 13
3.2 Dyadic Permutations 14
3.3 Cyclic Dyadic Permutations 16
3.4 Rotationless Dyadic Permutations 18

4 Transitive Homeomorphisms of I^n and R^n 22
4.1 Transitive Homeomorphisms 22
4.2 A Transitive Homeomorphism of I^n 23
4.3 A Transitive Homeomorphism of R^n 24
4.4 Topological Weak Mixing 25
4.5 A Chaotic Homeomorphism of I^n 27
4.6 Periodic Approximations 29

5 Fixed Points and Area Preservation 31
5.1 Introduction 31
5.2 The Plane Translation Theorem 32
5.3 The Open Square 33
5.4 The Torus 35
5.5 The Annulus 36

6 Measure Preserving Lusin Theorem 38
6.1 Introduction 38
6.2 Approximation Techniques 41
6.3 Proof of Theorem 6.2(i) 45

7 Ergodic Homeomorphisms 48
7.1 Introduction 48
7.2 A Classical Proof of Generic Ergodicity 50

8 Uniform Approximation in $\mathcal{G}[I^n, \lambda]$ and Generic Properties in $\mathcal{M}[I^n, \lambda]$ 53
8.1 Introduction 53
8.2 Rokhlin Towers and Stochastic Matrices 55

Part II: Measure Preserving Homeomorphisms of a Compact Manifold 59
9 Measures on Compact Manifolds 61
9.1 Introduction to Part II 61
9.2 General Measures on the Cube 61
9.3 Manifolds 64
9.4 Measures on Compact Manifolds 66
9.5 Typical Properties in $\mathcal{M}[X, \mu]$ 69

10 Dynamics on Compact Manifolds 71
10.1 Introduction 71
10.2 Genericity Results for Manifolds 71
10.3 Applications to Fixed Point Theory 75

Part III: Measure Preserving Homeomorphisms of a Noncompact Manifold 79
11 Introduction to Part III 81
11.1 Noncompact Manifolds 81
11.2 Topologies on $\mathcal{G}[X, \mu]$ and $\mathcal{M}[X, \mu]$: Noncompact Case 81
11.3 Main Results for Sigma Compact Manifolds 84

11.4 Outline of Part III 86

12 Ergodic Volume Preserving Homeomorphisms of R^n 89
12.1 Introduction 89
12.2 Homeomorphisms of R^n with Invariant Cubes 90
12.3 Generic Ergodicity in $\mathcal{M}[R^n, \lambda]$ 93
12.4 Other Typical Properties in $\mathcal{M}[R^n, \lambda]$ 94

13 Manifolds Where Ergodicity Is Not Generic 98
13.1 Introduction 98
13.2 Two Examples 98
13.3 Ends of a Manifold: Informal Introduction 102
13.4 Another Look at R^n 104
13.5 The Flip on the Strip 104
13.6 The Flip on Manhattan 104
13.7 Shear Map on the Strip 105

14 Noncompact Manifolds and Ends 106
14.1 Introduction 106
14.2 End Compactification 106
14.3 Examples of End Compactifications 107
14.4 Algebra $\mathcal{Q}$ of Clopen Sets 108
14.5 Measures on Ends 109
14.6 Compact Separating Sets 112
14.7 End Preserving Lusin Theorem 113
14.8 Induced Homeomorphism h^* 115
14.9 The Charge Induced by a Homeomorphism 121
14.10 h-moving Separating Sets 126
14.11 End Conditions for Homeomorphic Measures 128

15 Ergodic Homeomorphisms: The Results 130
15.1 Introduction 130
15.2 Consequences of Theorem 15.1 132

16 Ergodic Homeomorphisms: Proofs 137
16.1 Introduction 137
16.2 Outline of Proofs of Theorems 15.1 and 15.2 138
16.3 Proof of Theorem 15.1: Strip Manifold 140
16.4 Proofs of Theorems 15.1 and 15.2: General Case 143

17 Other Properties Typical in $\mathcal{M}[X, \mu]$ 154
17.1 A General Existence Result 154

17.2 Proof of Theorem 17.1 155
17.3 Weak Mixing End Homeomorphisms 157
17.4 Maximal Chaos on Noncompact Manifolds 158

Appendix 1 **Multiple Rokhlin Towers and Conjugacy Approximation** 160
A1.1 Introduction 160
A1.2 Skyscraper Constructions 161
A1.3 Multiple Tower Rokhlin Theorem 166
A1.4 Pointwise Conjugacy Approximation 174
A1.5 Specified Transition Probabilities 177
A1.6 Setwise Conjugacy Approximation 179
A1.7 Infinite Measure Constructions 183

Appendix 2 **Homeomorphic Measures** 188
A2.1 Introduction 188
A2.2 Homeomorphic Measures on the Cube 189
A2.3 Homeomorphic Measures on Compact Manifolds 195
A2.4 Homeomorphic Measures on Noncompact Manifolds 196
A2.5 Proof of the Berlanga–Epstein Theorem 198

Bibliography 205
Index 213