Contents

Chapter 1. Introduction 1
1.1 Overview 1
1.2 Examples of Piecewise Monotonic Transformations and the Density Functions of Absolutely Continuous Invariant Measures 2
Chapter 2. Preliminaries 7
2.1 Review of Measure Theory 7
2.2 Spaces of Functions and Measures 9
2.3 Functions of Bounded Variation in One Dimension 16
2.4 Conditional Expectations 26
Problems for Chapter 2 27
Chapter 3. Review of Ergodic Theory 29
3.1 Measure-Preserving Transformations 29
3.2 Recurrence and Ergodicity 31
3.3 The Birkhoff Ergodic Theorem 40
3.4 Mixing and Exactness 49
3.5 The Spectrum of the Koopman Operator and the Ergodic Properties of τ 52
3.6 Basic Constructions of Ergodic Theory 57
3.7 Infinite and Finite Invariant Measures 67
Problems for Chapter 3 68
Chapter 4. The Frobenius–Perron Operator 74
4.1 Motivation 74
4.2 Properties of the Frobenius–Perron Operator 77
4.3 Representation of the Frobenius–Perron Operator 85
Problems for Chapter 4 87
Chapter 5. Absolutely Continuous Invariant Measures 96
5.1 Introduction 96
5.2 Existence of Absolutely Continuous Invariant Measures 96
5.3 Lasota–Yorke Example of a Transformation without Absolutely Continuous Invariant Measure 102

5.4 Rychlik's Theorem for Transformations with Countably Many Branches 106
Problems for Chapter 5 108

Chapter 6. Other Existence Results 110
6.1 The Folklore Theorem 110
6.2 Rychlik's Theorem for $C^{1+\varepsilon}$ Transformations of the Interval 118
6.3 Piecewise Convex Transformations 121
Problems for Chapter 6 125

Chapter 7. Spectral Decomposition of the Frobenius–Perron Operator 127
7.1 Theorem of Ionescu–Tulcea and Marinescu 127
7.2 Quasi-Compactness of Frobenius–Perron Operator 128
7.3 Another Approach to Spectral Decomposition: Constrictiveness 135
Problems for Chapter 7 138

Chapter 8. Properties of Absolutely Continuous Invariant Measures 139
8.1 Preliminary Results 139
8.2 Support of an Invariant Density 140
8.3 Speed of Convergence of the Iterates of $P_\tau^n f$ 147
8.4 Bernoulli Property 148
8.5 Central Limit Theorem 151
8.6 Smoothness of the Density Function 162
Problems for Chapter 8 166

Chapter 9. Markov Transformations 174
9.1 Definitions and Notation 174
9.2 Piecewise Linear Markov Transformations and the Matrix Representation of the Frobenius–Perron Operator 175
9.3 Eigenfunctions of Matrices Induced by Piecewise Linear Markov Transformations 178
9.4 Invariant Densities of Piecewise Linear Markov Transformations 180
9.5 Irreducibility and Primitivity of Matrix Representations of Frobenius–Perron Operators 183

9.6 Bounds on the Number of Ergodic Absolutely Continuous Invariant Measures 191
9.7 Absolutely Continuous Invariant Measures that Are Maximal 198
Problems for Chapter 9 207

Chapter 10. Compactness Theorem and Approximation of Invariant Densities 209
10.1 Introduction 209
10.2 Strong Compactness of Invariant Densities 210
10.3 Approximation by Markov Transformations 216
10.4 Application to Matrices: Compactness of Eigenvectors for Certain Non-Negative Matrices 221

Chapter 11. Stability of Invariant Measures 226
11.1 Stability of a Linear Stochastic Operator 226
11.2 Deterministic Perturbations of Piecewise Expanding Transformations 232
11.3 Stochastic Perturbations of Piecewise Expanding Transformations 238
Problems for Chapter 11 250

Chapter 12. The Inverse Problem for the Frobenius–Perron Equation 252
12.1 The Ershov–Malinetskii Result 252
12.2 Solving the Inverse Problem by Matrix Methods 254

Chapter 13. Applications 260
13.1 Application to Random Number Generators 260
13.2 Why Computers Like Absolutely Continuous Invariant Measures 262
13.3 A Model for the Dynamics of a Rotary Drill 270
13.4 A Dynamic Model for the Hipp Pendulum Regulator 282
13.5 Control of Chaotic Systems 286
13.6 Kołodziej's Proof of Poncelet's Theorem 303
Problems for Chapter 13 309

Solutions to Selected Problems 310

Bibliography 381

Index 395