

Contents

Part I. Dissipative Structures

Introduction	3
1. A Representative Example of Dissipative Structure	5
1.1 Bénard Convection	5
1.2 The Belousov–Zhabotinskii Reaction	14
2. Amplitude Equations and Their Applications	21
2.1 The Newell–Whitehead Equation and the Stability of Periodic Solutions	21
2.2 Anisotropic Fluids and the Ginzburg–Pitaevskii Equation ...	26
2.3 Topological Defects and Their Motion	29
2.4 The Amplitude Equation of an Oscillating Field	34
2.5 The Properties of the Complex Ginzburg–Landau Equation ..	36
2.5.1 Propagating Planar Wave Solutions and Their Stability	36
2.5.2 Rotating Spiral Waves	38
2.5.3 Hole Solutions and Disordered Patterns	39
3. Reaction–Diffusion Systems and Interface Dynamics	43
3.1 Interfaces in Single-Component Bistable Systems	43
3.2 Solitary Wave Pulses and Periodic Wave Pulse Trains in Excitable Systems	47
3.3 Spiral Waves in Excitable Systems	52
3.4 Multiple Spiral Waves and the Turing Pattern	58
3.4.1 Compound Spiral Rotation	58
3.4.2 The Turing Pattern	59
3.5 The Instability of Interfaces and Formation of Structure	62
4. Phase Dynamics	69
4.1 Weak Turbulence of Periodic Structures and the Phase Equation	69
4.2 Phase Waves and Phase Turbulence of Oscillating Fields ...	76
4.2.1 The Phase Equation of an Oscillating Field and Its Applications	76

4.2.2	Phase Waves and the Target Pattern.....	78
4.2.3	Phase Turbulence	82
4.3	The Phase Dynamics of Interfaces	84
4.4	Multiple Field Dynamics	86
5.	Foundations of Reduction Theory.....	93
5.1	Two Simple Examples.....	93
5.2	The Destabilization of Stationary Solutions	98
5.3	Foundations of the Amplitude Equation	100
5.4	The Introduction of Continuous Spatial Degrees of Freedom .	107
5.4.1	The Hopf Bifurcation	107
5.4.2	The Turing Instability	109
5.5	Fundamentals of Phase Dynamics	112
5.5.1	Phase Dynamics in a Uniform Oscillating Field	113
5.5.2	Phase Dynamics for a System with Periodic Structure.	114
5.5.3	Interface Dynamics in a Two-Dimensional Medium ...	115
 Supplement I:		
	Dynamics of Coupled Oscillator Systems.....	119
SI.1	The Phase Dynamics of a Collection of Oscillators.....	119
SI.2	Synchronization Phenomena	121
<hr/>		
Part II. The Structure and Physics of Chaos		
<hr/>		
Introduction		127
6. A Physical Approach to Chaos		129
6.1	The Phase Space Structure of Dissipative Dynamical Systems	129
6.2	The Phase Space Structure of Conservative Dynamical Systems	134
6.3	Orbital Instability and the Mixing Nature of Chaos	139
6.3.1	The Liapunov Number	139
6.3.2	The Expansion Rate of Nearby Orbits, $\lambda_1(X_t)$	141
6.3.3	Mixing and Memory Loss.....	144
6.4	The Statistical Description of Chaos	145
6.4.1	The Statistical Stability of Chaos.....	145
6.4.2	Time Coarse-Graining and the Spectrum $\psi(\Lambda)$	146
6.4.3	The Statistical Structure of Chaos	148
7. Bifurcation Phenomena of Dissipative Dynamical Systems		151
7.1	Band Chaos of the Hénon Map	151
7.2	The Derivation of Several Low-Dimensional Maps	155
7.2.1	The Hénon Map.....	157

7.2.2	The Annulus Map	157
7.2.3	The Standard Map ($J = 1$)	159
7.2.4	One-Dimensional Maps ($J = 0$)	160
7.3	Bifurcations of the One-Dimensional Quadratic Map	161
7.3.1	2^n -Bifurcations and 2^n -Band Bifurcations	161
7.3.2	The Self-Similarity and Renormalization Transformation of 2^n -Bifurcations	166
7.3.3	The Similarity of 2^n -Band Bifurcations	170
7.4	Bifurcations of the One-Dimensional Circle Map	174
7.4.1	Phase-Locked Band Chaos	174
7.4.2	Phase-Unlocked Fully Extended Chaos	177
8.	The Statistical Physics of Aperiodic Motion	183
8.1	The Statistical Structure Functions of the Coarse-Grained Orbital Expansion Rate	183
8.1.1	The Baker Transformation	185
8.1.2	Attractor Destruction in the Quadratic Map	187
8.1.3	Attractor Merging in the Circle Map	189
8.1.4	Bifurcations of the Hénon Map	192
8.1.5	The Slopes s_α and s_β of $\psi(\Lambda)$	193
8.2	The Singularity Spectrum $f(\alpha)$	195
8.2.1	The Multifractal Dimension $D(q)$	198
8.2.2	Partial Local Dimensions $\alpha_1(X)$ and $\alpha_2(X)$	198
8.2.3	$f(\alpha)$ Spectra of Critical Attractors	199
8.3	Theory Regarding the Slope of $\psi(\Lambda)$	203
8.3.1	The Slope s_α Due to the Folding of W^u for Tangency Structure	203
8.3.2	The Slope s_β Resulting from Collision with the Saddle S	205
8.4	The Relation Between $f(\alpha)$ and $\psi(\Lambda)$	209
8.4.1	The Linear Segment of $f(\alpha)$ Resulting from the Folding of W^u in the Presence of Tangency Structure	212
8.4.2	The Linear Segment of $f(\alpha)$ Caused by Bifurcation	214
9.	Chaotic Bifurcations and Critical Phenomena	217
9.1	Crisis and Energy Dissipation in the Forced Pendulum	217
9.1.1	The Slope s_δ Induced by the Cantor Repellor	218
9.1.2	The Spectrum $\psi(W)$ of the Energy Dissipation Rate ..	222
9.1.3	The Formation of the Attractor Form in Figure 6.1 ...	224
9.2	Fully-Extended Chaos That Exists After Attractor Merging ..	224
9.2.1	Attractor Merging in the Annulus Map	225
9.2.2	Attractor Merging in the Forced Pendulum	227

9.3	Critical Phenomena and Dynamical Similarity of Chaos	230
9.3.1	The Self-Similar Time Series of Critical Attractors	230
9.3.2	The Algebraic Structure Functions of the Critical Attractor	233
9.3.3	The Internal Similarity of Bands for the Spectrum $\psi(\Lambda)$	235
9.3.4	The Form Characterizing the Disappearance of Two-Dimensional Fractality	238
10.	Mixing and Diffusion in Chaos of Conservative Systems	241
10.1	The Dynamical Self-Similarity of the Last KAM Torus	241
10.1.1	The Self-Similar F_m Time Series	242
10.1.2	The Symmetric Spectrum $\psi_\beta(\beta)$	242
10.2	The Mixing of Widespread Chaos	243
10.2.1	The Form of $\psi(\Lambda)$ and the Breaking of Time-Reversal Symmetry	245
10.2.2	The Appearance of Anomalous Scaling Laws for Mixing	247
10.3	Anomalous Diffusion Due to Islands of Accelerator Mode Tori	250
10.3.1	Accelerator Mode Periodic Orbits	250
10.3.2	Long-Time Velocity Correlation	251
10.3.3	The Anomalous Nature of the Statistical Structure of the Coarse-Grained Velocity	252
10.4	Diffusion and Mixing of Fluids as a Result of Oscillation of Laminar Flow	254
10.4.1	Islands of Accelerator Mode Tori Existing Within Turnstiles	254
10.4.2	Anomalous Mixing Due to Long-Time Correlation	257
Supplement II:		
	On the Structure of Chaos	261
SII.1	On-Off Intermittency	261
SII.2	Anomalous Diffusion Induced by an Externally Applied Force	262
SII.3	Transport Coefficients and the Liapunov Spectrum	263
Summary of Part II		265

A. Appendix	269
A.1 Periodic Points of Conservative Maps and Their Neighborhoods	269
A.2 Variance and the Time Correlation Function	271
A.3 The Cantor Repellor of Intermittent Chaos	271
Bibliography	279
Index	295