

Contents

Introduction	vii
1. Finite-dimensional situation	viii
2. Infinite dimensional systems (the problem and the result)	xi
3. Applications	xvi
4. Remarks on averaging theorems	xxv
5. Remarks on nearly integrable symplectomorphisms	xxv
6. Notations	xxvii
Part 1. Symplectic structures and Hamiltonian systems in scales of Hilbert spaces	1
1.1. Symplectic Hilbert scales and Hamiltonian equations	1
1.2. Canonical transformations	6
1.3. Local solvability of Hamiltonian equations	9
1.4. Toroidal phase space	11
1.5. A version of the former constructions	12
Part 2. Statement of the main theorem and its consequences	13
2.1. Statement of the main theorem	14
2.2. Reformulation of Theorem 1.1.	18
2.3. Nonlinear Schrödinger equation	22
2.4. Schrödinger equation with random potential	28
2.5. Nonlinear Schrödinger equation on the real line	33
2.6. Nonlinear string equation	35
2.7. On non-commuting operators J , A and partially hyperbolic invariant tori	39
Appendix. On superposition operator in Sobolev spaces	44

Part 3. Proof of the main theorem	45
3.1. Preliminary transformations	45
3.2. Proof of Theorem 1.1	53
3.3. Proof of Lemma 2.2 (solving the homological equations)	67
3.4. Proof of Lemma 3.1 (estimation of the small divisors)	73
3.5. Proof of Lemma 2.3 (estimation of the change of variables)	78
3.6. Proof of Refinement 2	82
3.7. On reducibility of variational equations	84
3.8. Proof of Theorem 1.2	85
Appendix A. Interpolation theorem	91
Appendix B. Some estimates for Fourier series	92
Appendix C. Lipschitz homeomorphisms of Borel sets	92
Appendix D. Cauchy estimate	93
List of notations	94
Bibliography	96
Index	101