

Contents

0. Introduction	1
1. Equations of Motion and the Hamiltonian Formalism	9
1.1 The Hamiltonian Formalism for Waves in Continuous Media	9
1.1.1 The Hamiltonian in Normal Variables	9
1.1.2 Interaction Hamiltonian for Weak Nonlinearity	15
1.1.3 Dynamic Perturbation Theory. Elimination of Nonresonant Terms	18
1.1.4 Dimensional Analysis of the Hamiltonian Coefficients	21
1.2 The Hamiltonian Formalism in Hydrodynamics	25
1.2.1 Clebsh Variables for Ideal Hydrodynamics	25
1.2.2 Vortex Motion in Incompressible Fluids	29
1.2.3 Sound in Continuous Media	29
1.2.4 Interaction of Vortex and Potential Motions in Compressible Fluids	31
1.2.5 Waves on Fluid Surfaces	33
1.3 Hydrodynamic-Type Systems	37
1.3.1 Langmuir and Ion-Sound Waves in Plasma	37
1.3.2 Atmospheric Rossby Waves and Drift Waves in Inhomogeneous Magnetized Plasmas	43
1.4 Spin Waves	51
1.4.1 Magnetic Order, Energy and Equations of Motion ..	51
1.4.2 Canonical Variables	53
1.4.3 The Hamiltonian of a Heisenberg Ferromagnet	54
1.4.4 The Hamiltonian of Antiferromagnets	56
1.5 Universal Models	58
1.5.1 Nonlinear Schrödinger Equation for Envelopes	59
1.5.2 Kadomtsev-Petviashvili Equation for Weakly Dispersive Waves	60
1.5.3 Interaction of Three Wave Packets	61
2. Statistical Description of Weak Wave Turbulence	63
2.1 Kinetic Wave Equation	63
2.1.1 Equations of Motion	63

2.1.2	Transition to the Statistical Description	64
2.1.3	The Three-Wave Kinetic Equation	66
2.1.4	Applicability Criterion of the Three-Wave Kinetic Equation (KE)	67
2.1.5	The Four-Wave Kinetic Equation	70
2.1.6	The Quantum Kinetic Equation	72
2.2	General Properties of Kinetic Wave Equations	75
2.2.1	Conservation Laws	75
2.2.2	Boltzmann's H-Theorem and Thermodynamic Equilibrium	78
2.2.3	Stationary Nonequilibrium Distributions	80
3.	Stationary Spectra of Weak Wave Turbulence	83
3.1	Kolmogorov Spectra of Weak Turbulence in Scale-Invariant Isotropic Media	83
3.1.1	Dimensional Estimations and Self-Similarity Analysis	84
3.1.2	Exact Stationary Solutions of the Three-Wave Kinetic Equation	86
3.1.3	Exact Stationary Solutions for the Four-Wave Kinetic Equations	93
3.1.4	Exact Power Solutions of the Boltzmann Equation .	101
3.2	Kolmogorov Spectra of Weak Turbulence in Nearly Scale-Invariant Media	102
3.2.1	Weak Acoustic Turbulence	102
3.2.2	Media with Two Types of Interacting Waves	108
3.3	Kolmogorov Spectra of Weak Turbulence in Anisotropic Media	117
3.3.1	Stationary Power Solutions	117
3.3.2	Fluxes of Integrals of Motion and Families of Anisotropic Power Solutions	120
3.4	Matching Kolmogorov Distributions with Pumping and Damping Regions	123
3.4.1	Matching with the Wave Source	124
3.4.2	Influence of Dissipation	135
4.	The Stability Problem and Kolmogorov Spectra	145
4.1	The Linearized Kinetic Equation and Neutrally Stable Modes	145
4.1.1	The Linearized Collision Term	145
4.1.2	General Stationary Solutions and Neutrally Stable Modes	147
4.2	Stability Problem for Kolmogorov Spectra of Weak Turbulence	156
4.2.1	Perturbation of the Kolmogorov Spectrum	160

4.2.2	Behavior of Kolmogorov-Like Turbulent Distributions. Stability Criterion	173
4.2.3	Physical Examples	184
4.3	Nonstationary Processes and the Formation of Kolmogorov Spectra	190
4.3.1	Analysis of Self-Similar Substitutions	191
4.3.2	Method of Moments	197
4.3.3	Numerical Simulations	200
5.	Physical Applications	207
5.1	Weak Acoustic Turbulence	207
5.1.1	Three-Dimensional Acoustics with Positive Dispersion: Magnetic Sound and Phonons in Helium	209
5.1.2	Two-Dimensional Acoustics with Positive Dispersion: Gravity-Capillary Waves on Shallow Water and Waves in Flaky Media	218
5.1.3	Nondecay Acoustic Turbulence: Ion Sound, Gravity Waves on Shallow Water and Inertio-Gravity Waves	227
5.2	Wave Turbulence on Water Surfaces	229
5.2.1	Capillary Waves on Deep Water	229
5.2.2	Gravity Waves on Deep Water	230
5.2.3	Capillary Waves on Shallow Fluids	232
5.3	Turbulence Spectra in Plasmas, Solids, and the Atmosphere	233
5.3.1	Langmuir Turbulence in Isotropic Plasmas	233
5.3.2	Optical Turbulence in Nonlinear Dielectrics and Turbulence of Envelopes	236
5.3.3	Spin Wave Turbulence in Magnetic Dielectrics	237
5.3.4	Anisotropic Spectra in Plasmas	239
5.3.5	Rossby Waves	242
6.	Conclusion	245
A.	Appendix	249
A.1	Variational Derivatives	249
A.2	Canonicity Conditions of Transformations	250
A.3	Elimination of Nonresonant Terms from the Interaction Hamiltonian	252
	References	257
	Subject Index	263