

# Contents

	Preface	ix
<b>CHAPTER 1</b>	<b>FIRST EXAMPLES</b>	
	1. The Simplest Examples	1
	2. Linear Systems with Constant Coefficients	9
	Notes	13
<b>CHAPTER 2</b>	<b>NEWTON'S EQUATION AND KEPLER'S LAW</b>	
	1. Harmonic Oscillators	15
	2. Some Calculus Background	16
	3. Conservative Force Fields	17
	4. Central Force Fields	19
	5. States	22
	6. Elliptical Planetary Orbits	23
	Notes	27
<b>CHAPTER 3</b>	<b>LINEAR SYSTEMS WITH CONSTANT COEFFICIENTS AND REAL EIGENVALUES</b>	
	1. Basic Linear Algebra	29
	2. Real Eigenvalues	42
	3. Differential Equations with Real, Distinct Eigenvalues	47
	4. Complex Eigenvalues	55
<b>CHAPTER 4</b>	<b>LINEAR SYSTEMS WITH CONSTANT COEFFICIENTS AND COMPLEX EIGENVALUES</b>	
	1. Complex Vector Spaces	62
	2. Real Operators with Complex Eigenvalues	66
	3. Application of Complex Linear Algebra to Differential Equations	69
<b>CHAPTER 5</b>	<b>LINEAR SYSTEMS AND EXPONENTIALS OF OPERATORS</b>	
	1. Review of Topology in $\mathbb{R}^n$	75
	2. New Norms for Old	77
	3. Exponentials of Operators	82
	4. Homogeneous Linear Systems	89
	5. A Nonhomogeneous Equation	99
	6. Higher Order Systems	102
	Notes	108

<b>CHAPTER 6</b>	<b>LINEAR SYSTEMS AND CANONICAL FORMS OF OPERATORS</b>	
	1. The Primary Decomposition	110
	2. The $S + N$ Decomposition	116
	3. Nilpotent Canonical Forms	122
	4. Jordan and Real Canonical Forms	126
	5. Canonical Forms and Differential Equations	133
	6. Higher Order Linear Equations	138
	7. Operators on Function Spaces	142
<b>CHAPTER 7</b>	<b>CONTRACTIONS AND GENERIC PROPERTIES OF OPERATORS</b>	
	1. Sinks and Sources	144
	2. Hyperbolic Flows	150
	3. Generic Properties of Operators	153
	4. The Significance of Genericity	158
<b>CHAPTER 8</b>	<b>FUNDAMENTAL THEORY</b>	
	1. Dynamical Systems and Vector Fields	159
	2. The Fundamental Theorem	161
	3. Existence and Uniqueness	163
	4. Continuity of Solutions in Initial Conditions	169
	5. On Extending Solutions	171
	6. Global Solutions	173
	7. The Flow of a Differential Equation	174
	Notes	178
<b>CHAPTER 9</b>	<b>STABILITY OF EQUILIBRIA</b>	
	1. Nonlinear Sinks	180
	2. Stability	185
	3. Liapunov Functions	192
	4. Gradient Systems	199
	5. Gradients and Inner Products	204
	Notes	209
<b>CHAPTER 10</b>	<b>DIFFERENTIAL EQUATIONS FOR ELECTRICAL CIRCUITS</b>	
	1. An $RLC$ Circuit	211
	2. Analysis of the Circuit Equations	215
	3. Van der Pol's Equation	217
	4. Hopf Bifurcation	227
	5. More General Circuit Equations	228
	Notes	238
<b>CHAPTER 11</b>	<b>THE POINCARÉ-BENDIXSON THEOREM</b>	
	1. Limit Sets	239
	2. Local Sections and Flow Boxes	242
	3. Monotone Sequences in Planar Dynamical Systems	244

4.	The Poincaré–Bendixson Theorem	248
5.	Applications of the Poincaré–Bendixson Theorem	250
	Notes	254
<b>CHAPTER 12</b>	<b>ECOLOGY</b>	
1.	One Species	255
2.	Predator and Prey	258
3.	Competing Species	265
	Notes	274
<b>CHAPTER 13</b>	<b>PERIODIC ATTRACTORS</b>	
1.	Asymptotic Stability of Closed Orbits	276
2.	Discrete Dynamical Systems	278
3.	Stability and Closed Orbits	281
<b>CHAPTER 14</b>	<b>CLASSICAL MECHANICS</b>	
1.	The $n$ -Body Problem	287
2.	Hamiltonian Mechanics	290
	Notes	295
<b>CHAPTER 15</b>	<b>NONAUTONOMOUS EQUATIONS AND DIFFERENTIABILITY OF FLOWS</b>	
1.	Existence, Uniqueness, and Continuity for Nonautonomous Differential Equations	296
2.	Differentiability of the Flow of Autonomous Equations	298
<b>CHAPTER 16</b>	<b>PERTURBATION THEORY AND STRUCTURAL STABILITY</b>	
1.	Persistence of Equilibria	304
2.	Persistence of Closed Orbits	309
3.	Structural Stability	312
<b>AFTERWORD</b>		319
<b>APPENDIX I</b>	<b>ELEMENTARY FACTS</b>	
1.	Set Theoretic Conventions	322
2.	Complex Numbers	323
3.	Determinants	324
4.	Two Propositions on Linear Algebra	325
<b>APPENDIX II</b>	<b>POLYNOMIALS</b>	
1.	The Fundamental Theorem of Algebra	328
<b>APPENDIX III</b>	<b>ON CANONICAL FORMS</b>	
1.	A Decomposition Theorem	331
2.	Uniqueness of $S$ and $N$	333
3.	Canonical Forms for Nilpotent Operators	334

<b>APPENDIX IV THE INVERSE FUNCTION THEOREM</b>	<b>337</b>
<b>REFERENCES</b>	<b>340</b>
<b>ANSWERS TO SELECTED PROBLEMS</b>	<b>343</b>
Subject Index	355