Contents

1.1 Quantization

Preface

Chapter I		
Introduction	and	Motivation

1.2 Schrödinger Equation	2
1.3 Quantum Mechanics and Diffusion Processes	4
1.4 Equivalence of Schrödinger and Diffusion Equations	6
1.5 Time Reversal and Duality	7
1.6 QED and Quantum Field Theory	ç
1.7 What is the Schrödinger Equation?	10
1.8 Mathematical Contents	12
Chapter II Diffusion Processes and their Transformations	
2.1 Time Homogeneous Diffusion Processes	13
2.2 Time Inhomogeneous Diffusion Processes	17
2.3 Brownian Motions	19
2.4 Stochastic Differential Equations	21
2.5 Transformation by a Multiplicative Functional	27
2.6 Feynman-Kac Formula	31
2.7 Kac's Semi-Group and its Renormalization	33
2.8 Time Change	37
2.9 Dirichlet Problem	40
2.10 Feller's One-Dimensional Diffusion Processes	42
2.11 Feller's Test	46

Chapter III Duality and Time Reversal of Diffusion Processes

3.1 Kolmogoroff's Duality	55
3.2 Time Reversal of Diffusion Processes	63
3.3 Duality of Time-Inhomogeneous Diffusion Processes	69
3.4 Schrödinger's and Kolmogoroff's Representations	75
3.5 Some Remarks	86
Chapter IV	
Equivalence of Diffusion and	
Schrödinger Equations	
44.0	00
4.1 Change of Variable Formulae	89
4.2 Equivalence Theorem	94
4.3 Discussion of the Non-Linear Dependence	96
4.4 A Solution to Schrödinger's Conjecture	100
4.5 A Unified Theory	101
4.6 On Quantization	104
4.7 As a Diffusion Theory	107
4.8 Principle of Superposition	108
4.9 Remarks	114
Chapter V	
Variational Principle	
5.1 Problem Setting in <i>p</i> -Representation	115
5.2 Csiszar's Projection Theorem	118
5.3 Reference Processes	119
5.4 Diffusion Processes in Schrödinger's Representation	125
5.5 Weak Fundamental Solutions	131
5.6 An Entropy Characterization of the Markov Property	132
5.6 Remarks	135

Chapter VI **Diffusion Processes in q-Representation**

6.1	A Multiplicative Functional	139
6.2	Flows of Distribution Densities	146
6.3	Discussions on the q -Representation	149
6.4	What is the Feynman Integral?	152
6.5	A Remark on Kac's Semi-Group	156
6.6	A Typical Case	159
6.7	Hydrogen Atom	160
6.8	A Remark on $\{\mu_a, \mu_b\}$	161
	er VII	
Segre	gation of a Population	
7.1	Introduction	163
7.2	Harmonic Oscillator	164
7.3	Segregation of a Finite-System of Particles	166
7.4	A Formulation of the Propagation of Chaos	170
7.5	The Propagation of Chaos	171
7.6	Skorokhod Problem with Singular Drift	174
7.7	A Limit Theorem	185
7.8	A Proof of Theorem 7.1	195
7.9	Schrödinger Equations with Singular Potentials	198
	er VIII	
	Schrödinger Equation can Boltzmann Equation	
	-	
	Large Deviations	207
	The Propagation of Chaos in Terms of Large Deviations	215
8.3	Statistical Mechanics for Schrödinger Equations	218
8.4	Some Comments	220

Chapter IX	
Applications of the Statistical Model	
for Schrödinger Equations	
9.1 Segregation of a Monkey Population	223
9.2 An Eigenvalue Problem	226
9.3 Septation of Escherichia Coli	227
9.4 The Mass Spectrum of Mesons	233
9.5 Titius-Bode Law	238
Chapter X	
Relative Entropy and Csiszar's Projection	
10.1 Relative Entropy	239
10.2 Csiszar's Projection	243
10.3 Exponential Families and Marginal Distributions	246
Chapter XI	
Large Deviations	
11.1 Lemmas	253
11.2 Large Deviations of Empirical Distributions	255
Chapter XII	
Non-Linearity Induced by	
he Branching Property	
12.1 Branching Property	261
12.2 Non-Linear Equations of Branching Processes	265
12.3 Quasi-Linear Parabolic Equations	268
12.4 Branching Markov Processes with Non-Linear Drift	272
12.5 Revival of a Markov Process	273

12.6 Construction of Branching Markov Processes

276

Appendix:

a.1 Fényes' "Equation of Motion" of Probability Densities	281
a.2 Stochastic Mechanics	283
a.3 Segregation of a Population	287
a.4 Euclidean Quantum Mechanics	288
a.5 Remarks	290
a.6 Bohmian Mechanics	291
References	
Index	311