
Contents

Preface

Introduction 1

Chapter 1. Solitary waves and evolution equations 7

- 1.1. Solitary waves: a brief review 7
- 1.2. Evolution equations 16
 - 1.2.1. Introduction 16
 - 1.2.2. The iterative method 18
 - 1.2.3. The asymptotic (reductive perturbation) method 21
 - 1.2.4. The spectral method 23
 - 1.2.5. Comparison of the methods 26
 - 1.2.6. Remarks 33
- 1.3. Definitions 35

Chapter 2. Nonlinear conservative systems 44

- 2.1. The KdV equation 44
- 2.2. Inverse Scattering Transform (IST) 51
- 2.3. Formation of solitons from initial data 60
 - 2.3.1. Reflectionless potential 61
 - 2.3.2. General form of a potential 73
 - 2.3.3. Special forms of a potential 79
- 2.4. Discussion 91
 - 2.4.1. Velocity of KdV solitons 91
 - 2.4.2. Splitting of solitons 94

Chapter 3. Nonlinear systems perturbed by energy influx 98

- 3.1. Models 99
 - 3.1.1. Physical description 99
 - 3.1.2. Isolating the perturbed KdV equation 102

CONTENTS

3.2.	Asymptotic analysis	107
3.2.1.	Perturbative IST	107
3.2.2	Different types of energy influx	112
3.2.3.	Direct perturbation	122
3.3.	Numerical analysis	128
3.3.1.	Computer simulation of amplitude equations	128
3.3.2.	Stationary waves	134
3.3.3.	Transient waves	141
Chapter 4.	Nonlinear systems with essential energy influx	155
4.1.	Models	156
4.1.1.	Physical description	156
4.1.2.	The Hodgkin - Huxley model	158
4.1.3.	The Lieberstein model	163
4.1.4.	The FitzHugh - Nagumo model	165
4.1.5.	Other models	168
4.2.	Evolution equations	171
4.2.1.	The hyperbolic model and the basic evolution equation	171
4.2.2.	Full evolution equations	175
4.2.3.	General remarks	180
4.3.	Properties of evolution equations	183
4.3.1.	Preliminary	183
4.3.2.	Stationary waves, general analysis	184
4.3.3.	Stationary waves, numerical results	206
4.3.4.	Transient waves	218
4.4.	Improved models	224
4.4.1.	Dynamic nonlinearity	225
4.4.2.	The recovery variable revisited	229
4.4.3.	The van der Pol equation revisited	240
4.4.4.	Development of models	246
4.5.	Inverse problems	251
Closing remarks	258	
Appendix: FFT technique for evolution equations (T. Peipman)	262	
References	267	
Index	278	