

# Contents

<b>List of Tables</b>	xiii
<b>List of Figures</b>	xv
<b>List of Abbreviations</b>	xvii
<b>List of Symbols</b>	xix
<b>1 Introduction</b>	1
1.1 Motivation . . . . .	2
1.2 Research Objectives . . . . .	3
1.3 Outline . . . . .	4
<b>I Foundations of Stochastic Inventory Control</b>	7
<b>2 Basic Inventory Management</b>	9
2.1 Inventory Management and Control . . . . .	10
2.2 Motivations for Holding Inventory . . . . .	11
2.3 Classification of Inventories . . . . .	13
2.3.1 The Strategic Perspective . . . . .	13

2.3.2	The Operational Perspective . . . . .	14
2.4	Inventory-related Costs . . . . .	16
2.4.1	Order Cost . . . . .	16
2.4.2	Holding Cost . . . . .	17
2.4.3	Penalty Cost . . . . .	18
2.5	Service Level . . . . .	19
<b>3</b>	<b>Stochastic Inventory Control</b>	<b>21</b>
3.1	Characteristics of Inventory Models . . . . .	22
3.1.1	Management Decisions . . . . .	22
3.1.2	System-Inherent Characteristics . . . . .	23
3.2	Types of Inventory Control Policies . . . . .	25
3.3	Periodic Inventory Control . . . . .	27
3.3.1	Single-Period Inventory Control . . . . .	28
3.3.2	Multi-Period Inventory Control . . . . .	29
3.3.3	Service-Constraint Multi-Period Inventory Control . . . . .	32
<b>II</b>	<b>Essential Stochastic Processes</b>	<b>33</b>
<b>4</b>	<b>Markov Chains</b>	<b>35</b>
4.1	The Markov Property . . . . .	36
4.2	Homogeneous Markov Chains . . . . .	37
4.3	Limit Distribution and Invariant Distribution . . . . .	38
4.4	Communication Classes . . . . .	40
4.5	Class Properties . . . . .	41
4.5.1	Aperiodicity . . . . .	41
4.5.2	Recurrency and Transiency . . . . .	41

<b>CONTENTS</b>	<b>ix</b>
4.5.3 Class Criteria . . . . .	43
4.6 Uniqueness of Limit Distributions . . . . .	44
4.7 Multi-Dimensional Markov Chains . . . . .	45
4.8 Applications of Markov Chains . . . . .	46
<b>5 Numerical Solution of Markov Chains</b>	<b>47</b>
5.1 System Reduction Approaches . . . . .	48
5.1.1 Reduced System Approaches . . . . .	48
5.1.2 Geometric Tail Distributions . . . . .	49
5.2 State Space Reduction Approaches . . . . .	50
5.2.1 Augmentations . . . . .	50
5.2.2 Pointwise Convergence . . . . .	52
5.3 Algorithms . . . . .	54
5.3.1 Sheskin's Partitioning Algorithm . . . . .	54
5.3.2 Power Iteration . . . . .	56
5.4 Criteria for Choosing a Specific Algorithm . . . . .	56
<b>6 Comparing Stochastic Processes</b>	<b>59</b>
6.1 Stochastic Ordering for Distribution Functions . . . . .	59
6.2 The Sample Path Method . . . . .	61
<b>III Stochastic Inventory Control with Customer Segmentation</b>	<b>63</b>
<b>7 Introduction to Inventory Rationing</b>	<b>65</b>
7.1 Examples for Different Customer Classes . . . . .	66
7.2 Related Areas . . . . .	67
7.3 Rationing Rules . . . . .	69

7.4	Backorders and Backorder Clearing Mechanisms . . . . .	71
7.5	Literature Review . . . . .	73
7.5.1	Characterizations of the Optimal Policy . . . . .	73
7.5.2	Evaluations and Optimizations of Critical Level Policies	76
7.5.3	Other Studies Involving Critical Level Rationing . . .	80
7.6	Classification of Our Work . . . . .	81
<b>8</b>	<b>Modeling Approach</b>	<b>83</b>
8.1	Modeling Framework . . . . .	84
8.2	Recursive Expressions for Backorders . . . . .	84
8.3	Markov Chain . . . . .	86
8.4	Structural Results . . . . .	87
8.5	Proofs . . . . .	90
<b>9</b>	<b>Prioritization by Penalty Costs</b>	<b>97</b>
9.1	Optimization Approach . . . . .	98
9.1.1	Convexity of Objective Function . . . . .	98
9.1.2	Transition Matrix . . . . .	100
9.1.3	Cost Function . . . . .	101
9.1.4	Numerical Optimization Algorithm . . . . .	102
9.2	Numerical Results . . . . .	102
9.2.1	Critical Level Policy versus Benchmark Policies . . .	103
9.2.2	Combination Heuristic . . . . .	106
9.3	Conclusion . . . . .	106
9.4	Proofs . . . . .	108
<b>10</b>	<b>Prioritization by Service Levels</b>	<b>111</b>
10.1	Analytical Insights . . . . .	112

10.1.1	Service Level Constraints . . . . .	112
10.1.2	Potentially Optimal Parameter Constellations . . . . .	115
10.1.3	Structural Results for the Optimal Solution . . . . .	117
10.1.4	Cost Function . . . . .	119
10.1.5	Model Alteration: $\beta$ -Service Levels . . . . .	120
10.2	Numerical Results . . . . .	121
10.2.1	Optimization Algorithm . . . . .	122
10.2.2	Critical Level Policy versus Benchmark Policies . . . . .	123
10.2.3	Heuristic Approach . . . . .	126
10.3	Conclusion . . . . .	127
10.4	Proofs . . . . .	129
<b>11</b>	<b>Dynamic Rationing Policies</b>	<b>135</b>
11.1	The Next Period Optimization Policy . . . . .	136
11.1.1	The Policy . . . . .	136
11.1.2	Structural results . . . . .	137
11.2	The Linear Critical Level Policy . . . . .	138
11.2.1	The Policy . . . . .	138
11.2.2	Structural results . . . . .	139
11.3	Efficiency of the Dynamic Rationing Policies . . . . .	142
11.3.1	Next Period Optimization and Constant Rationing Policy . . . . .	143
11.3.2	Linear Critical Level Policy and Constant Rationing Policy . . . . .	147
11.3.3	Next Period Optimization and Linear Critical Level Policy . . . . .	150
11.4	Conclusion . . . . .	152
11.5	Proofs . . . . .	153

<b>12 Conclusion and Critical Review</b>	<b>157</b>
12.1 Contributions . . . . .	157
12.2 Critical Review . . . . .	159
12.3 Future Research . . . . .	160
<b>Bibliography</b>	<b>163</b>

# List of Tables

3.1 Basic inventory control policies and costs . . . . .	27
6.1 Distribution of variables X and Y . . . . .	60
7.1 Critical level rationing for two customer classes . . . . .	71
7.2 Relations between queueing theory and Ha's model . . . . .	75
9.1 Critical level policy vs. benchmark policies - penalty cost . .	105
9.2 Heuristic vs. critical level policy and aggregation policy - penalty cost . . . . .	107
10.1 Optimality sets . . . . .	117
10.2 Critical level policy vs. benchmark policies - service level . .	125
10.3 Heuristic vs. critical level policy and aggregation policy - service level . . . . .	128
11.1 Sensitivity of the total cost per period under LCL . . . . .	141
11.2 Optimal weight factors for different class 1 penalty costs and lead times . . . . .	142
11.3 Next period optimization policy vs. constant rationing policy	144

11.4 Expected backorders and inventory at the end of a period under CRP and NPO . . . . .	145
11.5 Linear critical level policy vs. constant rationing policy . . .	149

# List of Figures

1.1	Structure of research . . . . .	5
2.1	A simple inventory system . . . . .	11
2.2	Types of inventory from different perspectives . . . . .	16
2.3	Categories of inventory-related costs . . . . .	19
3.1	Inventory level and inventory position under $(R, S)$ -policy . .	31
4.1	Example for a Markov chain . . . . .	38
7.1	A simple divergent multi-echelon system . . . . .	68
7.2	Research on critical level policies with positive lead time . .	82
8.1	Effect of $S$ on $EB_1$ , $EB_2$ and $EIH$ for given $CL$ . . . . .	89
8.2	Effect of $CL$ on $EB_1$ , $EB_2$ , and $EIH$ for given $S$ . . . . .	90
9.1	Concept of diagonals . . . . .	99
9.2	Expected total cost per period along different diagonals . . .	103
10.1	Feasible and potentially optimal $(S, CL)$ -pairs . . . . .	114
10.2	Optimal solutions . . . . .	116

10.3 Clustered optimal solutions depending on service level requirements . . . . .	118
10.4 Expected optimal costs per period depending on $\alpha_2$ for fixed $\alpha_1$ . . . . .	120
10.5 The sets $A_1$ and $A_2(\alpha_2)$ . . . . .	131
11.1 Distribution of the rationing level $CL_t^{NPO}$ . . . . .	138
11.2 Savings under LCL and a policy with exponential critical level compared to CRP . . . . .	140
11.3 Distribution of the rationing level $CL_t^{LCL}$ . . . . .	141
11.4 Savings of NPO compared to CRP for different class 1 penalty costs . . . . .	146
11.5 Savings of LCL compared to CRP for different class 1 penalty costs . . . . .	150
11.6 Difference between savings under LCL and NPO for different class 1 penalty costs . . . . .	151
11.7 When to apply which rationing policy . . . . .	153