

Contents

Chapter One Introduction

1.1	Historical background and outline of contents	1
1.2	Asymptotic theory of linear differential equations	4

Chapter Two Meijer functions and integral function theory

2.1	Mellin-Barnes integral solutions of the generalised hypergeometric equation	
2.1.1	The solutions of the generalised hypergeometric equation	14
2.1.2	Mellin-Barnes integral representations of the Meijer G-function	16
2.1.3	The convergence of Mellin-Barnes integrals	18
2.1.4	Evaluation of the Mellin-Barnes integral for $G_{p,q}^{m,n}(z)$	23
2.2	Asymptotic properties of Meijer's function	
2.2.1	The algebraic asymptotic expansion of $G_{p,q}^{m,n}(z)$	27
2.2.2	The asymptotic expansion of $G_{p,q}^{q,0}(z)$ and $G_{p,q}^{q,1}(z)$	31
2.2.3	A brief discussion of the asymptotic behaviour of $G_{p,q}^{m,0}(z)$	35
2.3	The asymptotic expansion of integral functions of the hypergeometric type	
2.3.1	Introduction	36
2.3.2	The Maclaurin coefficients	39
2.3.3	An integral representation of ${}_pF_q(z)$	42
2.3.4	The algebraic and exponential expansions	46
2.3.5	The asymptotic expansion of ${}_pF_q(z)$ for $ z \rightarrow \infty$	50
2.3.6	Examples	52

3.1	The fundamental hypergeometric function solutions	56
3.2	The solution $U_{n,p}(z)$ for $1 \leq p < n$	60
3.3	The asymptotic expansion of $U_{n,p}(z)$ for large $ z $ ($1 \leq p < n$)	64
3.4	The recurrence relation for the coefficients c_m	67
3.5	The special cases of $U_{n,p}(z)$ corresponding to $p = 1, 2$	
3.5.1	The case $p = 1$	73
3.5.2	The case $p = 2$	78
3.6	The exponentially small solution $V_{n,p}(z)$ ($1 \leq p < n$)	84
3.6.1	Definition and expansion of $V_{n,p}(z)$ in the principal sector	85
3.6.2	The expansion theorem for $V_{n,p}(z)$ in the z -plane	87
3.6.3	Examples	91
3.7	The algebraic solution $W_{n,p}(\beta_j; z)$ ($1 \leq p < n$)	95
3.7.1	Definition and expansion of $W_{n,p}(\beta_1; z)$ in the principal sector	95
3.7.2	The expansion theorem for $W_{n,p}(\beta_1; z)$ in the z -plane	97
3.7.3	The solutions $W_{n,p}(\beta_j; z)$ for $j = 1, 2, \dots, p$	100
3.7.4	Other algebraic solutions	101
3.8	Even and odd solutions when n is even	102
3.8.1	The even and odd solutions $E_{n,p}(z)$ and $O_{n,p}(z)$	103
3.8.2	The asymptotic expansions of $E_{n,p}(z)$ and $O_{n,p}(z)$ for $ z \rightarrow \infty$	106
3.8.3	The even and odd solutions $E_{n,p}(z)$ and $O_{n,p}(z)$ in the logarithmic case ($p = 2$)	109
3.9	The solutions $U_{n,p}(z)$ and $W_{n,p}(\beta_j; z)$ when $n = p$	111
3.9.1	Analytic continuation outside $ z = 1$	112
3.9.2	Examples	115
3.10	Integral representations	120
3.10.1	Laplace integral representations of $U_{n,p}(z)$ for $n \geq p \geq 1$	121
3.10.2	Integral representations of $V_{n,p}(z)$ and $W_{n,p}(\beta_j; z)$ when $p = 1$ and $p = 2$	126
3.10.3	Multiple integral representation of $U_{n,p}(z)$	130
3.10.4	Mellin integral representations	134
3.11	Examples	136

Chapter Four The equation with positive integer $\beta = n$

4.1	The fundamental hypergeometric function solutions	148
4.2	The solution $U_{n,p}^m(z)$ for $0 \leq p < n$	
4.2.1	Definition and expansion of $U_{n,p}^m(z)$ in the principal sector	150
4.2.2	The expansion theorem in the z -plane	152
4.2.3	Laplace integral representation of $U_{n,p}^m(z)$	156
4.3	Examples of sectorial behaviour	159
4.4	The exponentially small solution $V_{n,p}^m(z)$	168
4.5	The algebraic solution $W_{n,p}^m(\beta_j; z)$	172
4.6	The even and odd solutions $E_{n,p}^m(z)$ and $O_{n,p}^m(z)$	
4.6.1	The case n, m both even	175
4.6.2	The case n, m both odd	180
4.7	The solutions $U_{n,p}^m(z)$ and $W_{n,p}^m(\beta_j; z)$ when $n = p$	182
4.8	Examples	186

Chapter Five The equation with arbitrary real β and generalisations

5.1	The case of general real β	196
5.1.1	The algebraic and exponential type solutions $W_{n,p}^\beta(\beta_j; z)$ and $V_{n,p}^\beta(z)$	196
5.1.2	The formally self-adjoint equations of even and odd order	202
5.2	Formal methods for more general equations	209
5.2.1	Leading asymptotic behaviour in (5.1.1) by the WKB method	209
5.2.2	WKB method for equations with general power coefficients	213
5.2.3	WKB method for equations with polynomial coefficients	218
5.3	Full asymptotic expansion of solutions of a fourth order equation with a polynomial coefficient	222
5.3.1	Integral representations of the solutions	223
5.3.2	The asymptotic expansion of $u_{\lambda,\mu}(a, z)$ for large $ z $	227
5.3.3	Exponentially small and algebraic solutions	235

Chapter Six	Application to physical problems	
6.1	The resistive tearing mode in magnetohydrodynamics	241
6.1.1	Introduction and the resistive tearing mode boundary layer equations	241
6.1.2	Solutions of an equation of order $2n$ with a linear inhomogeneity	245
6.1.3	The solution of the boundary layer equations	249
6.1.4	The tearing mode in the long wavelength limit	251
6.1.5	The tearing mode in toroidal geometry	255
6.2	Stationary convective-like modes in a plasma slab	
6.2.1	Introduction and formulation of the eigenvalue problem	258
6.2.2	The solution of the boundary layer equation and matching	261
6.3	The stability of stellar winds	265
6.4	Some boundary layer problems in fluid dynamics	269
6.4.1	A laminar jet in a viscous rotating fluid	271
6.4.2	Slow viscous flow of a stratified fluid past a flat plate	276
6.5	Linear wave conversion at the lower hybrid resonance	282
6.6	Wave propagation due to a steadily moving source on a floating ice sheet	288
Chapter Seven	Application to spectral theory	292
7.1	Some problems in spectral theory of differential operators	293
7.2	A theorem on L^2 solutions	299
7.3	Applications to even order operators	304
7.3.1	A fourth order equation with quadratic coefficient	305
7.3.2	Equations with power coefficients	308
7.4	Applications to odd order operators	322
7.5	The Titchmarsh-Weyl coefficients for fourth order equations	326
7.5.1	Method of construction for the general fourth order equation	327
7.5.2	The Titchmarsh-Weyl coefficients for (7.5.1)	329
7.5.3	Asymptotic behaviour of $m_{i,j}(\lambda)$ for large $ \lambda $	331
References		333