

Contents of Part I

Ordinary Differential Equations The One-Dimensional Theory $x' = f(t, x)$

| | |
|---|------|
| Series Preface for Texts in Applied Mathematics | v |
| Preface | vii |
| Acknowledgments | xi |
| Ways to Use This Book | xiii |
| Introduction | 1 |
| Chapter 1 Qualitative Methods | 11 |
| 1.1 Field of Slopes and Sketching of Solutions | 11 |
| 1.2 Qualitative Description of Solutions | 19 |
| 1.3 Fences | 23 |
| 1.4 Funnels and Antifunnels | 30 |
| 1.5 The Use of Fences, Funnels, and Antifunnels | 34 |
| 1.6 Vertical Asymptotes | 42 |
| Exercises | 49 |
| Chapter 2 Analytic Methods | 67 |
| 2.1 Separation of Variables | 68 |
| 2.2 Linear Differential Equations of First Order | 71 |
| 2.3 Variation of Parameters | 76 |
| 2.4 Bank Accounts and Linear Differential Equations | 79 |
| 2.5 Population Models | 81 |
| 2.6 Exact Differential Equations | 85 |
| 2.7 Series Solutions of Differential Equations | 92 |
| Exercises | 98 |

| | | |
|------------------------------|---|-----|
| Chapter 3 | Numerical Methods | 111 |
| 3.1 | Euler's Method | 111 |
| 3.2 | Better Numerical Methods | 118 |
| 3.3 | Analysis of Error, According to Approximation Method | 125 |
| 3.4 | Finite Accuracy | 133 |
| 3.5 | What to Do in Practice | 143 |
| | Exercises | 149 |
| Chapter 4 | Fundamental Inequality, Existence, and Uniqueness | 157 |
| 4.1 | Existence: Approximation and Error Estimate | 157 |
| 4.2 | Uniqueness: The Leaking Bucket Versus Radioactive Decay | 158 |
| 4.3 | The Lipschitz Condition | 165 |
| 4.4 | The Fundamental Inequality | 169 |
| 4.5 | Existence and Uniqueness | 174 |
| 4.6 | Bounds on Slope Error for Other Numerical Methods | 180 |
| 4.7 | General Fence, Funnel, and Antifunnel Theorems | 183 |
| | Exercises | 189 |
| Chapter 5 | Iteration | 197 |
| 5.1 | Iteration: Representation and Analysis | 199 |
| 5.2 | The Logistic Model and Quadratic Polynomials | 214 |
| 5.3 | Newton's Method | 223 |
| 5.4 | Numerical Methods as Iterative Systems | 235 |
| 5.5 | Periodic Differential Equations | 245 |
| 5.6 | Iterating in One Complex Dimension | 255 |
| | Exercises | 273 |
| Appendix. | Asymptotic Development | 297 |
| A1. | Equivalence and Order | 297 |
| A2. | Technicalities of Defining Asymptotic Expansions | 298 |
| A3. | First Examples; Taylor's Theorem | 301 |
| A4. | Operations on Asymptotic Expansions | 304 |
| A5. | Rules of Asymptotic Development | 306 |
| References | | 307 |
| Answers to Selected Problems | | 309 |
| Index | | 345 |