

Contents

1.	Introduction	1
2.	Dynamical Systems	7
2.1.	Particles	7
2.2.	Systems of Particles	9
2.3.	Forces and Laws of Motion	10
2.4.	Galilean Transformations	12
2.5.	Arguments of the Forces	15
2.6.	The Problems of Particle Mechanics	17
3.	Representations of the Motion	19
3.1.	The Configuration Space	19
3.2.	The Event Space	21
3.3.	The State Space	24
3.4.	The State-Time Space	26
3.5.	Notions on the Concept of Stability	26
3.6.	Problems	28
4.	Constraints	29
4.1.	General Observations	29
4.2.	Holonomic Constraints	31
4.3.	Nonholonomic Constraints	39
4.4.	The Pfaffian Forms	43
4.5.	When is a System of Constraints Holonomic?	45
4.6.	Accessibility (of the Configuration Space)	48
4.7.	Problems	51

5. The Strictly Newtonian Mechanics Problem	55
5.1. General Remarks	55
5.2. The Given Quantities and Relations	55
5.3. The First Problem	57
5.4. The Second Problem	59
5.5. Other Problems	59
5.6. Concluding Remarks	60
6. Some Rigid Body Kinematics	61
6.1. The Rigid Body	61
6.2. Finite Rotation	63
6.3. The Direction Cosines	66
6.4. Orthogonal Transformations	69
6.5. The Matrix Notation	71
6.6. Properties of the Rotation Matrix	73
6.7. The Composition of Rotations	75
6.8. Applications	76
(a) The Euler Angles	77
(b) The Rodrigues Formulas	82
6.9. Problems	84
7. Some Rigid Body Kinetics	87
7.1. Introductory Remarks	87
7.2. The Inertial Parameters in Rotated Axes	92
7.3. Angular Momentum and Principal Axes	94
7.4. The Ellipsoids of Cauchy and Poinsot	96
7.5. The General Motion of Rigid Bodies	107
7.6. Problems	111
8. The Nature of Lagrangean Mechanics	115
8.1. General Remarks	115
8.2. The Generalizations by Lagrange	115
9. Virtual Displacement and Virtual Work	119
9.1. General Observations	119
9.2. Classification of Displacements	119
9.3. D'Alembert's Principle	121
9.4. The Nature of the Forces of Constraint	128
9.5. The Virtual Velocity	139
9.6. The Variation	145

9.7. Possible Velocities and Accelerations	147
9.8. The Fundamental Equation	149
9.9. The Nature of the Given Forces	150
9.10. Given Forces Which Are Functions of Constraint Forces	152
9.11. Problems	157
10. Hamilton's Principle	161
10.1. The Kinetic Energy	161
10.2. Kinetic Energy in Catastrophic Systems	162
10.3. The Energy Relations in Catastrophic Systems	163
10.4. The Central Principle	167
10.5. The Principle of Hamilton	169
10.6. Noncontemporaneous Variations	174
10.7. Lagrange's Principle of Least Action	176
10.8. Jacobi's Principle of Least Action	179
10.9. Problems	181
11. Generalized Coordinates	185
11.1. Introductory Remarks	185
11.2. The Theory of Generalized Coordinates	186
11.3. The Nature of Generalized Coordinates	190
11.4. The δ Operator for Generalized Coordinates	194
11.5. Exceptional Cases	195
11.6. Problems	199
12. The Fundamental Equation in Generalized Coordinates	201
12.1. The Kinetic Energy	201
12.2. Two Equalities	204
12.3. The Fundamental Equation	204
12.4. Generalized Potential Forces	206
12.5. Velocity-Dependent Potentials	207
12.6. Problems	209
13. Lagrange's Equations	211
13.1. The Dynamical Problem	211
13.2. The Multiplier Rule	212
13.3. Derivation from the Fundamental Equation	214
13.4. Derivation from the Central Principle	216
13.5. Derivation from Hamilton's Principle	217
13.6. Dynamic Coupling and Decoupling	221

13.7. Special Forms of Lagrange's Equations	226
(a) Existence of a Potential	226
(b) Holonomic Systems	226
(c) Rayleigh's Dissipation Function	227
(d) The Dissipation Function of Lur'e	229
13.8. The Principle of Least Action Reconsidered	232
13.9. Problems	235
14. Embedding Constraints	239
14.1. Introductory Remarks	239
14.2. A Fallacy	239
14.3. Embedding of Nonholonomic Constraints	243
14.4. Problems	246
15. Formulating Problems by Lagrange's Equations	247
15.1. General Remarks	247
15.2. The Unconstrained Particle	249
15.3. The Holonomically Constrained Particle	253
15.4. The Nonholonomically Constrained Particle	256
15.5. Systems of Particles and Rigid Bodies	259
15.6. Problems	270
16. The Integration	273
16.1. The Meaning of an Integral	273
16.2. Jacobi's Integral	276
16.3. The Routhian Function and the Momentum Integrals	279
(a) The Legendre Transformation	280
(b) The Routhian Function	283
16.4. Partial and Complete Separation of Variables	286
16.5. Solution in Quadratures	290
16.6. Qualitative Integration	293
16.7. Problems	298
17. Stability	301
17.1. Introductory Remarks	301
17.2. Definition of Stability	302
17.3. The Variational Equations	304
17.4. Some Remarks on Indirect Methods	305
17.5. Some Remarks on Liapunov's Direct Method	311
(a) The Autonomous Case	312
(b) The Nonautonomous Case	319
17.6. Problems	320

18. Applications	323
18.1. Introductory Remarks	323
18.2. The Single Particle	324
18.3. Systems of Particles	328
18.4. Nonholonomic Systems	334
18.5. Problems	346
19. About Celestial Problems	349
19.1. Historical Notes	349
19.2. The Central Force Problem	351
19.3. The Central Force Problem Continued—The Apsides	354
19.4. The Central Force Problem Continued—On Bertrand's Theorem	356
19.5. The n -Body Problem	364
19.6. The Two-Body Problem	368
19.7. Some Information about the Three-Body Problem	369
19.8. Problems	370
20. Topics in Gyrodynamics	373
20.1. Introduction	373
20.2. The Heavy Symmetrical Top	374
20.3. The Gyroscope	383
20.4. The Gyrocompass	385
20.5. Problems	388
21. Impulsive Motion	391
21.1. General Remarks	391
21.2. The Fundamental Equation	393
21.3. Impulsive Constraints	395
21.4. The Fundamental Equation with Impulsive Constraints	400
21.5. Impulsive Motion Theorems	401
21.6. Lagrange's Equations for Impulsive Motion	411
21.7. Problems	412
BIBLIOGRAPHY	415
INDEX	417