

Contents

Chapter 1. A Mathematical and Historical Tour	1
1.1 Images from Dynamical Systems	1
1.2 A Brief History of Dynamics	5
Chapter 2. Examples of Dynamical Systems	9
2.1 An Example from Finance	9
2.2 An Example from Ecology	11
2.3 Finding Roots and Solving Equations	12
2.4 Differential Equations	15
Chapter 3. Orbits	17
3.1 Iteration	17
3.2 Orbits	18
3.3 Types of Orbits	19
3.4 Other Orbits	22
3.5 The Doubling Function	24
3.6 Experiment: The Computer May Lie	25
Chapter 4. Graphical Analysis	29
4.1 Graphical Analysis	29
4.2 Orbit Analysis	32
4.3 The Phase Portrait	33
Chapter 5. Fixed and Periodic Points	36
5.1 A Fixed Point Theorem	36
5.2 Attraction and Repulsion	37
5.3 Calculus of Fixed Points	38
5.4 Why Is This True?	42
5.5 Periodic Points	46
5.6 Experiment: Rates of Convergence	48

Chapter 6. Bifurcations	52
6.1 Dynamics of the Quadratic Map	52
6.2 The Saddle-Node Bifurcation	57
6.3 The Period-Doubling Bifurcation	61
6.4 Experiment: The Transition to Chaos	63
Chapter 7. The Quadratic Family	69
7.1 The Case $c = -2$	69
7.2 The Case $c < -2$	71
7.3 The Cantor Middle-Thirds Set	75
Chapter 8. Transition to Chaos	82
8.1 The Orbit Diagram	82
8.2 The Period-Doubling Route to Chaos	89
8.3 Experiment: Windows in the Orbit Diagram	92
Chapter 9. Symbolic Dynamics	97
9.1 Itineraries	97
9.2 The Sequence Space	98
9.3 The Shift Map	103
9.4 Conjugacy	106
Chapter 10. Chaos	114
10.1 Three Properties of a Chaotic System	114
10.2 Other Chaotic Systems	121
10.3 Manifestations of Chaos	126
10.4 Experiment: Feigenbaum's Constant	128
Chapter 11. Sarkovskii's Theorem	133
11.1 Period 3 Implies Chaos	133
11.2 Sarkovskii's Theorem	137
11.3 The Period 3 Window	142
11.4 Subshifts of Finite Type	146
Chapter 12. The Role of the Critical Orbit	154
12.1 The Schwarzian Derivative	154
12.2 The Critical Point and Basins of Attraction	157
Chapter 13. Newton's Method	164
13.1 Basic Properties	164
13.2 Convergence and Nonconvergence	169
Chapter 14. Fractals	176
14.1 The Chaos Game	176

14.2	The Cantor Set Revisited	178
14.3	The Sierpinski Triangle	180
14.4	The Koch Snowflake	182
14.5	Topological Dimension	185
14.6	Fractal Dimension	186
14.7	Iterated Function Systems	190
14.8	Experiment: Iterated Function Systems	197
Chapter 15.	Complex Functions	203
15.1	Complex Arithmetic	203
15.2	Complex Square Roots	207
15.3	Linear Complex Functions	209
15.4	Calculus of Complex Functions	212
Chapter 16.	The Julia Set	221
16.1	The Squaring Function	221
16.2	The Chaotic Quadratic Function	226
16.3	Cantor Sets Again	227
16.4	Computing the Filled Julia Set	233
16.5	Experiment: Filled Julia Sets and Critical Orbits	238
16.6	The Julia Set as a Repellor	239
Chapter 17.	The Mandelbrot Set	246
17.1	The Fundamental Dichotomy	246
17.2	The Mandelbrot Set	249
17.3	Experiment: Periods of Other Bulbs	253
17.4	Experiment: Periods of the Decorations	257
17.5	Experiment: Find the Julia Set	258
17.6	Experiment: Spokes and Antennae	259
17.7	Experiment: Similarity of the Mandelbrot and Julia Sets	260
Chapter 18.	Further Projects and Experiments	263
18.1	The Tricorn	263
18.2	Cubics	264
18.3	Exponential Functions	267
18.4	Trigonometric Functions	270
18.5	Complex Newton's Method	273
Appendix A.	Mathematical Preliminaries	279
Appendix B.	Algorithms	287
Appendix C.	References	295
Index	299