

TABLE OF CONTENTS

Introduction	xi
List of Symbols	xiv
Part One: General Information.	
Chapter I. <u>The ring of germs of differentiable functions of n real variables.</u>	
1. The ring E_n of germs of functions of n variables	1
2. The maximal ideal and its powers	1
3. The algebra of jets	3
4. Ideals of finite codimension in E_n	3
5. The study of some other ideals of E_n	5
6. Remarks and examples	7
CHAPTER II. <u>The group of local diffeomorphisms of \mathbb{R}^n</u>	
1. The group L_n of local diffeomorphisms at the origin of \mathbb{R}^n	9
2. The group L_n^k of k -jets of local diffeomorphisms	9
3. The action of L_n and L_n^k on E_n and J_n^k	11
4. Infinitesimal generators of L_n and L_n^k	12
CHAPTER III. <u>Elements of the classifications of germs of functions of n variables.</u>	
1. Introduction	17
2. Morse's Lemma (first proof)	18
3. Morse's Lemma (second proof)	21
4. Generalization	24
5. Orbits of germs of finite codimension	26
6. Comments and examples	29

CHAPTER IV. Introduction to the study of deformations

1. Introduction and definitions
2. Tangent space and codimension of a germ of a scalar function
3. The notion of a universal deformation of a germ of finite codimension
4. The fundamental geometric lemma in the theory of deformations
5. The universal deformation of a Morse germ and the Decomposition Lemma.
6. Universal deformation of x^3

CHAPTER V. Generic singularities of mappings of the plane to the plane

1. Introduction
2. Folds and cusps for mappings of the plane to the plane.
3. Whitney's Theorem
4. Proof
5. Generic singularities of mappings from \mathbb{R}^s into \mathbb{R}^2 ($s \geq 2$)

CHAPTER VI. The division theorem of order two.

1. Introduction. Statement of the general division theorem
2. Reduction to a particular case: the canonical division theorem
3. Reduction of the division theorem of order two to a lemma of Whitney
4. Preliminaries for the proof of Whitney's lemma
5. Differentiable functions on real subsets of \mathbb{C}^n
6. Proof of Whitney's Lemma
7. Proof of Lemma 5.5

Appendix

CHAPTER VII. Thom's transversality theorem

1. Introduction and review
2. Sard's Theorem
3. Proof of Sard's Theorem
4. The fundamental lemma of transversality
5. First application: the generic behavior of the rank of differentiable mappings
6. Thom's Transversality Theorem
7. Application: generic singularities of mappings from \mathbb{R}^2 to \mathbb{R}^2

Part Two: The differentiable preparation theorem.

CHAPTER VIII. The importance of flat functions

1. Flat functions on a real vector subspace
2. The extension theorem of Lojasiewicz
3. A division lemma

CHAPTER IX. The division theorem

1. Introduction
2. The analytic case
3. The differentiable theorem of Newton
4. Proof of the canonical division theorem of order k .

CHAPTER X. The Malgrange-Mather preparation theorem

1. Introduction
2. Statement of the Malgrange-Mather Preparation Theorem
3. The division theorem as a special case of the preparation theorem
4. A first generalization
5. A second generalization
6. The preparation theorem in general

Part Three: Universal deformations of germs of real-valued functions

CHAPTER XI. Universal deformations of real-valued functions

1. The fundamental theorem of universal deformations
2. Universal deformations and transversality
3. Universal deformations of potentials
4. A remark concerning the Weierstrass Preparation Theorem

CHAPTER XII. Classification of germs of real-valued functions of dimension less than six; the elementary catastrophes of R. Thom

1. Preliminary remarks on ideals of finite codimension in E_n
2. Introduction to the classification of germs of codimension less than or equal to 5
3. Classification of germs of corank 1
4. Classification of germs of corank 2 (and codimension ≤ 5)
5. Geometric description of germs of codimension ≤ 5
6. A transversality theorem
7. The elementary catastrophes of R. Thom

Part Four: Singularities of differentiable mappings

CHAPTER XIII. Introduction to the local study of differential mappings. Tangent space.

1. Introduction. Definitions
2. Rank and unfoldings
3. Jets of mappings. Orbits
4. Tangent space
5. Tangent space to the orbit of a map germ
6. Algebraic description of the tangent space
7. Tangent spaces: examples of computations

CHAPTER XIV. Universal unfoldings

1. Introduction
2. The geometric lemma of the unfolding theory
3. The algebraic lemma of the unfolding theory
4. Proof of the universal unfolding theorem
5. Application: the singularities $\Sigma^{1, \dots, 1}$
of \mathbb{R}^{n+1} to \mathbb{R}^{n+1} ($n \geq 1$)

CHAPTER XV. Classification of stable map germs

1. Introduction
2. Stable unfoldings of germs of finite type
3. Characterization of map germs of finite type
4. The contact group; contact orbits
5. Stable map germs: the main geometric characterization
6. Description of the contact orbits

CHAPTER XVI. Classification of stable germs

1. The fundamental theorem and its consequences
2. Study of germs embedded in an unfolding
3. A fundamental lemma
4. Proof of the fundamental lemma
5. Orbits of stable germs

CHAPTER XVII. Generic singularities: examples

1. Introduction. Stability of generic singularities
2. The study of $K(p, 1, 1)$
3. Generic singularities of rank p from \mathbb{R}^{p+1} to \mathbb{R}^{p+1}
4. $K(p, 1, 2)$
5. Generic singularities of rank p for mappings from \mathbb{R}^{p+1} to \mathbb{R}^{p+2} ($p \geq 1$)
6. $K(p, 2, 1)$
7. Stability is not generic in $C^\infty(\mathbb{R}^9, \mathbb{R}^8)$

8. The study of some $K(s,s,2)$ germs, $s \geq 2$. 243
9. Singularities of mappings from \mathbb{R}^{2s} to \mathbb{R}^{s+2}
($s \geq 2$) 247

Bibliography 249

Index 253