Contents

PREFACE ix

Introduction LINEARITY—A SHANGRI-LA 1

I.1 Why Linearity? / 1
I.2 From Whence Linearity / 2
I.3 What is Linearity? / 2
I.4 Things Linear or Not Linear / 3
I.5 Our Shangri-La / 4

1 FOUR INTERESTING EQUATIONS 5

1.0 The Equations / 5
1.1 The Solutions—A Preview / 6
1.2 Solving Equation (i): $\dot{x} + x = 0$ / 6
1.3 Solving Equation (ii): $\dot{x} + |x| = 0$ / 10
1.4 Solving Equation (iii): $|\dot{x}| + x = 0$ / 12
1.5 Solving Equation (iv): $|\dot{x}| + |x| = 0$ / 12
1.6 A Variation on Equation (iv): $|\dot{x}| = |x|$ / 12
1.7 Existence/Uniqueness of Solutions / 14
1.8 Problems / 21
1.9 References and Related Literature / 21

2 ANALYTIC SOLUTIONS TO NONLINEAR DIFFERENTIAL EQUATIONS 23

2.0 Introduction / 23
2.1 First-Order Base Equations / 26
2.2 Second-Order Base Equations / 29
2.3 The Ricatti Equation / 34
2.4 Nonlinear Base Equations / 36
2.5 Derivative and Integral Functional Relations / 43
2.6 Matrix Nonlinear Differential Equations / 48
2.7 Application to the Calculus of Variations / 51

2.8 Problems / 56
2.9 References and Related Literature / 59

3 LINEARIZATION **61**

3.0 Introduction / 61
3.1 Linearizing Algebraic Functions / 61
3.2 Linearizing a Transistor / 64
3.3 Linearization of Differential Functions / 66
3.4 Linearizing Satellite Motion / 76
3.5 Concluding Example / 83
3.6 Summary / 88
3.7 Problems / 88
3.8 References and Related Literature / 93

4 THE DESCRIBING FUNCTION **94**

4.1 Describing Function / 94
4.2 Frequency-Dependent Describing Functions / 109
4.3 Digital Simulation Verifies Describing Function Analysis / 113
4.4 Asymmetric Describing Functions / 115
4.5 Problems / 122
4.6 References / 126

5 SOME PROPERTIES OF NONLINEAR SYSTEMS **127**

5.0 Introduction / 127
5.1 Linear System Characteristics / 127
5.2 Nonlinear Equations with Periodic Solutions / 138
5.3 Limit Cycles / 141
5.4 Nonlinear System Behavior / 144
5.5 Some Physically Realizable Nonlinearities / 165
5.6 Problems / 175
5.7 References and Related Literature / 178

6 LIAPUNOV STABILITY **179**

6.0 Introduction / 179
6.1 Liapunov Stability: An Overview / 180
6.2 Construction of Liapunov Functionals / 191
6.3 The Lur'e Problem / 194
6.4 The Popov Criterion / 211
6.5 Problems / 214
6.6 References and Related Literature / 216

7 RECURSIONS AND THEIR STABILITY 217

7.1 Recursions / 217
7.2 The Mechanic's Rule / 220
7.3 Singularities and Peculiarities / 227
7.4 The Logistics Map / 231
7.5 Recursing Differential Equations / 245
7.6 Problems / 248
7.7 References / 253

8 DIGITAL SIMULATION 254

8.0 Background / 254
8.1 Recursion Formulae: Fundamental to Digital Simulation / 254
8.2 The Sampling Process Creates Discrete Data / 264
8.3 An Approach to Digital Simulation: Introduce Samplers / 265
8.4 Concept of Pulse Filters Implicit in Digital Simulations / 269
8.5 Introducing *Z*-Transforms / 270
8.6 The Pulse Filter Now Becomes a *Z*-Transform / 272
8.7 Introductory Example of Digital Simulation / 272
8.8 Digitally Simulating a Feedback System / 274
8.9 Simulating Hysteresis Due to Backlash in Gears / 277
8.10 Simulating a System with Hysteresis / 279
8.11 Problems / 284
8.12 References / 284

9 SPREADSHEET SIMULATION—A TUTORIAL 285

9.1 Introduction / 285
9.2 Use of the "Copy" Command / 286
9.3 Using the Graph Command / 288
9.4 Summary / 291
9.5 Some Heuristic Exercises—Brief Descriptions / 292
9.6 Suggested Solutions to the Exercises / 293
9.7 References / 322

10 AN ISOBARIC CABIN PRESSURE CONTROL 323

10.0 Introduction / 323
10.1 Background / 324
10.2 How the Cabin is Pressurized / 325
10.3 Basic Numbers and Constraints / 325
10.4 Cabin Dynamics / 326
10.5 Design Evolution / 327
10.6 A Nonlinear Digital Simulation / 337
10.7 Limiting Cabin Pressure Rate / 342
10.8 Initializing Integrators / 342
10.9 Problems / 343
10.10 References and Related Literature / 345

INDEX 347