

Contents of Volume 2

<i>Preface</i>	xv
<i>Acknowledgements</i>	xviii
6 Models based on second order difference equations	1
6.1 <i>Some origins of maps in R^2</i> : Delayed and coupled logistic maps; Poincaré surface of section in extended phase space ($R^2 \times I$); area-preserving maps; nonconservative vs conservative maps; Levinson–Smith relaxation oscillator; Hénon and Heiles Hamiltonian	1
6.2 <i>Rotation and winding numbers</i> : Maps and flows; knots, algebraic constants of the motion	9
6.3 <i>The Cartwright–Littlewood, Levinson and Levi analyses</i> : The extraordinary family of solutions, K_0 , of the forced self-exciting oscillator; equivalence to Bernoulli sequences; Levi’s extensions	16
6.4 <i>Some abstract nonconservative maps in R^2</i> : Hénon’s map; strange attractor; contracting map; geometrically wild vs dynamically-wild sets; Lyapunov exponents; period-three does not imply chaos in R^2 ; a fractal boundary between basins of attraction; Julia and Mandelbrot sets; Newton map	22
6.5 <i>The standard map; twist maps</i> : Ball on a vibrating plate; the microtron accelerator; harmonic lattice in a periodic potential; toroidal magnetic fields; twist maps; characteristic multipliers	33
6.6 <i>‘Near-integrable’ systems</i> : Poincaré’s last geometric theorem; hyperbolic and elliptic fixed point pairs; stable and unstable manifolds; homoclinic and heteroclinic points; KAM surfaces; Poincaré’s chaotic tangle; Bernoulli sequence	41
6.7 <i>The breakup of KAM curves</i> : Small divisors, irrational rotation numbers; computer study; continued fraction representation; the standard map and the golden mean	57
6.8 <i>Physical regularity in mathematical chaos</i> : Chaotic magnetic field lines and differentiable magnetic fields	65
6.9 <i>Chirikov’s resonance-overlap criterion</i> : A periodically kicked oscillator example	67
6.10 <i>The numerical Poincaré map and discontinuous dynamics</i> : Hénon’s integration method	72
6.11 <i>The Hénon–Heiles and Toda–Hamiltonian systems</i> : Poincaré maps for a nonintegrable and integrable system	74

6.12	<i>Abstract area-preserving maps on R^2 and T^2</i> : Hénon's map; involutive maps; Arnold's cat map; mixing	80
6.13	<i>Maps of sets</i> : The folds and kinks of a periodically forced conservative oscillator	90
6.14	<i>Maps on a lattice</i> : Rannou's study of the standard map and generalization on a $N \times N$ lattice; the random-map ensemble; Arnold's cat map on a lattice; short Poincaré recurrence; ghosts	93
6.15	<i>Dynamic entropies and information production</i> : The shuffling of partitions in phase space; Kolmogorov–Sinai, and topological dynamic entropies; informational interpretation	101
6.16	<i>Epilogue</i> : order–order, order–chaos, chaos–chaos in the house!	118
	7 Models based on third order differential systems	125
7.1	<i>Linear third order equations</i> : Characterization of fixed points; stable, unstable, and center manifolds; various representations of flows	126
7.2	<i>Nonlinear flows</i> : Center cycles, saddle cycles; Poincaré's first return map; local Möbius band manifolds; some 'interacting' flows from neighboring saddle-nodes, or spiral nodes	132
7.3	<i>The Lorenz model</i> : Historical origin; bifurcation of the fixed points as a function of r ; contracting and global attracting character	138
7.4	<i>Lorenz chaotic dynamics</i> : The Lorenz 'map'; the Lorenz fractal 'mask', a strange attractor	145
7.5	<i>A 'Lorenz-dynamic' fluid system</i> : A fluid flow in circular tubes which obeys the Lorenz equations	151
7.6	<i>Dynamo dynamics</i> : The amazing dynamics of the Earth's magnetic field; a physically unrelated simple disk dynamo with similar chaotic dynamics (a Lorenz system)	155
7.7	<i>The Lorenz homoclinic and heteroclinic bifurcations</i> : The global topological properties of the flow, for r above the first homoclinic bifurcation, and below the first heteroclinic bifurcation – a highly convoluted picture; 'preturbulence'.	162
7.8	<i>The Lorenz–Hopf bifurcation</i> : The subcritical bifurcation	177
7.9	<i>Lorenz dynamics for various parameter values</i> : Stable limit cycles subharmonic (saddle-node) bifurcations; Möbius strips and knotted limit cycles; intermittencies; bistabilities; and much chaos!	178
7.10	<i>The Lyapunov exponents</i> : Definition and methods of obtaining them; application to the Lorenz system, and bistability	190
7.11	<i>Rössler's models</i> : The 'lifting' approach to modeling; model $R1$, Lyapunov exponents, Poincaré maps, Lorenz 'maps' and Cantor set; the 'walking stick' folding and second Cantor set; subharmonic bifurcations, largest Lyapunov exponent; 'funnel' attractor; phase coherence; the 'Dali' limit cycle; model $R2$, bistability chaos; the taffey-machine-on-a-lazy-susan flow	198
7.12	<i>Lyapunov exponents and the dimension of a strange attractor</i> : A heuristic discussion of possible relations; the Kaplan–Yorke conjecture	217
7.13	<i>Open systems – chemical oscillations</i> : The Belousov–Zhabotinskii discoveries; the Field–Noyes 'Oregonator' equations; relaxation oscillations	222

8 'Moderate-order' systems	231
8.1 <i>Linear systems</i> : Poincaré's variational equations; near fixed points, asymptotic properties (Lyapunov exponents), with periodic and quasi-periodic coefficients; lattice normal modes and Schrödinger's solution	232
8.2 <i>Turing's linear chemical morphogenesis system</i> : Instabilities in cellular chains which are coupled diffusively	240
8.3 <i>'Integrable' Hamiltonian systems</i> : Motion on an n -dimensional torus; separation of adjacent states	246
8.4 <i>The Kolmogorov–Arnold–Moser theorem</i> : 'Near-integrable' systems	253
8.5 <i>Poincaré's and Fermi's theorems; Arnold diffusion</i> .	256
8.6 <i>The Fermi–Pasta–Ulam phenomenon and equipartitioning</i> : The nonequipartitioning of energy; early example of 'synergetics'; equipartitioning <i>vis-à-vis</i> ergodicity; influence on irreversibility (lattice heat conduction)	259
8.7 <i>Molecular models</i> : Polynomial potentials; Toda's exponential potential	268
8.8 <i>Toda's solitary waves in a lattice</i> : Analytic solutions for one and two 'conserved' compression pulses; cnoidal waves	274
8.9 <i>The dynamics of various Toda lattices</i> : The Ford–Stoddard–Turner numerical prediction of the integrability of equal-mass Toda lattices; the Flaschka and Hénon integrals; 'physical' significance? diatomic Toda lattices ($1 \geq m_1/m_2 \geq 0$); influence on chaotic behavior and lattice heat conduction	281
8.10 <i>The Painlevé property and integrability conjecture</i> : Kovalevskaya's use of complex time to search for integrability	294
8.11 <i>Chemical oscillations and dissipative open-system structures</i> : The 'Brusselator' and diffusively coupled Brusselators (Turing structures)	304
8.12 <i>Smale's analysis of Turing's morphogenic system</i> : Generating 'life' from 'dead' components, using diffusion; global aspects	311
8.13 <i>Embedding the dynamics of high-order dissipative systems in lower dimensional R^n</i> : The method of Takens and Crutchfield, Farmer, Packard, and Shaw; torus-knot example; application to chemical oscillations and Taylor vortices	317
8.14 <i>Some dynamics of living systems</i> : Definition problems; Eigen and Schuster's hypercycle quasi-species, error catastrophe; replicator equations; mean fitness; general Lotka–Volterra equation; Hofbauer's theorem, topological orbital equivalence; Smale's observation	330
8.15 <i>Epilogue</i> : open systems; open sesame!	343
9 Solitaires: solitons and nonsolitons	348
9.1 <i>The continuum limit of lattices and 'solitaire' solutions</i>	350
9.2 <i>Riemann invariants and the Korteweg–de Vries (KdV) equation</i>	358
9.3 <i>A comparison of the Burgers and KdV equations</i> : Dissipation vs dispersion	361
9.4 <i>The exact solution of Burgers equation – The Hopf–Cole transformation</i>	365
9.5 <i>A brief history leading to the inverse scattering transform (IST)</i> : Zabusky and Kruskal's discovery of 'solitons'; conservation laws; Miura's transformation; the 'Schrödinger equation'	367

9.6	<i>The general solution of the KdV equation: Gardner–Green–Kruskal–Miura analysis; the Gel’fand–Levitan theorem; the inverse scattering transform</i>	373
9.7	<i>Pure soliton solutions: KdV equation; the Landau–Lifshitz subset</i>	381
9.8	<i>The Lax formulation: The KdV example</i>	387
9.9	<i>The sine–Gordon equation: Kinks; topological solitons; breather modes</i>	390
9.10	<i>Hirota’s ‘direct method’ in soliton theory: Hirota’s binary operators; nonlinear transformations (types A, B, and C); bilinear differential equation and special exact solution</i>	398
9.11	<i>The AKNS formulation of the IST: Ablowitz–Kaup–Newell–Segur extension of Zaharov and Shabat’s method; connection of IST general solutions with the sine–Gordon, sinh–Gordon, nonlinear Schrödinger, modified KdV, and Dym’s equations</i>	406
9.12	<i>Some Bäcklund transformations between difference equations: Bäcklund transformation; integrability condition; examples</i>	410
9.13	<i>Invariant Bäcklund transformations: Free parameter; nonlinear superposition; Riccati equation</i>	413
9.14	<i>Infinite number of conservation laws: Relationship to invariant Bäcklund transformation</i>	418
9.15	<i>Onward: Higher dimensions; resonant interactions</i>	420
	10 Coupled maps (CM) and cellular automata (CA)	427
10.1	<i>An overview: Lagrangian and Eulerian models, with continuous and discrete variables and functions</i>	427
10.2	<i>Some coupled maps (CM): Diffusive, and other coupling of cells obeying logistic-map dynamics; multiple-periodic regions; coexistence of many periods; ‘semi-periodic turbulence’; spatial-temporal intermittency; inhomogeneous CM</i>	431
10.3	<i>Coupled lattice maps (CLM = CA): General family; chemical turbulence model of Oono and Kohmoto; ‘turbulence’, ‘solitons’ and periodic space-time patterns; nonperiodic, excitable cells, diffusively coupled; bistable cells</i>	445
10.4	<i>General cellular automata (CA): Von Neumann’s question; Ulam’s suggestion; dynamics defined</i>	454
10.5	<i>‘Legal’ cellular automata: Quiescent and symmetry conditions</i>	457
10.6	<i>A general association for legal CA: Some possible physical relations to CA rules, states and configurations</i>	459
10.7	<i>Simple examples: ‘Self-reproduction’</i>	460
10.8	<i>Neighborhood configurations and dynamic rules: Rule number; ‘totalistic’ CA</i>	462
10.9	<i>Several classifications of CA dynamic properties: Four qualitative categories of dynamics</i>	464
10.10	<i>Entropies of one-dimensional CA: Some possibilities and shortcomings of entropy measurements of ‘chaos’ or ‘turbulence’</i>	473
10.11	<i>Particle-like dynamics from partial CA rules: Colliding ‘particles’ with and without delays; oscillating ‘molecule’</i>	478
10.12	<i>Two-dimensional CA: Von Neumann neighborhoods; Fredkin’s ‘self-reproducing’ rule</i>	482

10.13 <i>Garden-of-Eden configurations</i> : Configurations which cannot dynamical arise; Moore's theorem	484
10.14 <i>J.H. Conway's 'Life'</i> : Moore neighborhood; snakes, ponds, blinkers, beehives, barges, barberpoles, eaters, gliders and glider guns	487
10.15 <i>Excitable medium</i> : Quiescent, excited, and refractory states	491
10.16 <i>Invertible CA and physical dynamics</i> : Invertible <i>vis-à-vis</i> reversible dynamics; the description of nature, how can it best be achieved?	493
<i>Appendix</i> : von Neumann's questions	499
<i>Epilogue</i> : 'Understanding' complex systems: Order; organization; Endnote-models, causality, irreversibility	505
 <i>Appendixes</i>	
J On the Cartwright–Littlewood and Levinson studies of the forced relaxation oscillator	529
K Smale's horseshoe map	539
L Notes on the Kolmogorov–Arnold–Moser theorem	543
<i>Bibliography</i>	553
<i>Cumulative index (Volumes 1 and 2)</i>	622