

Contents

Preface	vii
Introduction	1
1. The Mechanics of Lagrange	5
1.1 Newton's Equations According to Lagrange	6
1.2 The Variational Principle of Lagrange	7
1.3 Conservation of Energy	10
1.4 Example: Space Travel in a Given Time Interval; Lambert's Formula	11
1.5 The Second Variation	14
1.6 The Spreading Trajectories	16
2. The Mechanics of Hamilton and Jacobi	19
2.1 Phase Space and Its Hamiltonian	19
2.2 The Action Function S	20
2.3 The Variational Principle of Euler and Maupertuis	22
2.4 The Density of Trajectories on the Energy Surface	23
2.5 Example: Space Travel with a Given Energy	26
3. Integrable Systems	30
3.1 Constants of Motion and Poisson Brackets	30
3.2 Invariant Tori and Action-Angle Variables	32
3.3 Multiperiodic Motion	34
3.4 The Hydrogen Molecule Ion	36
3.5 Geodesics on a Triaxial Ellipsoid	38
3.6 The Toda Lattice	41
3.7 Integrable versus Separable	43

4.	The Three-Body Problem: Moon–Earth–Sun	45
4.1	Reduction to Four Degrees of Freedom	45
4.2	Applications in Atomic Physics and Chemistry	48
4.3	The Action-Angle Variables in the Lunar Observations	50
4.4	The Best Temporary Fit to a Kepler Ellipse	53
4.5	The Time-Dependent Hamiltonian	56
5.	Three Methods of Solution	58
5.1	Variation of the Constants (Lagrange)	58
5.2	Canonical Transformations (Delaunay)	59
5.3	The Application of Canonical Transformations	62
5.4	Small Denominators and Other Difficulties	63
5.5	Hill’s Periodic Orbit in the Three-Body Problem	65
5.6	The Motion of the Perigee and the Node	70
5.7	Displacements from the Periodic Orbit and Hill’s Equation	72
6.	Periodic Orbits	75
6.1	Potentials with Circular Symmetry	77
6.2	The Number of Periodic Orbits in an Integrable System ...	80
6.3	The Neighborhood of a Periodic Orbit	82
6.4	Elliptic, Parabolic, and Hyperbolic Periodic Orbits	84
7.	The Surface of Section	87
7.1	The Invariant Two-Form	87
7.2	Integral Invariants and Liouville’s Theorem	89
7.3	Area Conservation on the Surface of Section	91
7.4	The Theorem of Darboux	93
7.5	The Conjugation of Time and Energy in Phase Space	95
8.	Models of the Galaxy and of Small Molecules	99
8.1	Stellar Trajectories in the Galaxy	100
8.2	The Hénon-Heiles Potential	102
8.3	Numerical Investigations	103
8.4	Some Analytic Results	106
8.5	Searching for Integrability with Kowalevskaya and Painlevé	109
8.6	Discrete Area-Preserving Maps	111
9.	Soft Chaos and the KAM Theorem	116
9.1	The Origin of Soft Chaos	116
9.2	Resonances in Celestial Mechanics	118

9.3	The Analogy with the Ordinary Pendulum	120
9.4	Islands of Stability and Overlapping Resonances	125
9.5	How Rational Are the Irrational Numbers?	129
9.6	The KAM Theorem	132
9.7	Homoclinic Points	135
9.8	The Lore of the Golden Mean	138
10.	Entropy and Other Measures of Chaos	142
10.1	Abstract Dynamical Systems	143
10.2	Ergodicity, Mixing, and K-Systems	145
10.3	The Metric Entropy	147
10.4	The Automorphisms of the Torus	149
10.5	The Topological Entropy	151
10.6	Anosov Systems and Hard Chaos	154
11.	The Anisotropic Kepler Problem	156
11.1	The Donor Impurity in a Semiconductor Crystal	156
11.2	Normalized Coordinates in the Anisotropic Kepler Problem	159
11.3	The Surface of Section	161
11.4	Construction of Stable and Unstable Manifolds	164
11.5	The Periodic Orbits in the Anisotropic Kepler Problem ..	168
11.6	Some Questions Concerning the AKP	171
12.	The Transition from Classical to Quantum Mechanics	173
12.1	Are Classical Mechanics and Quantum Mechanics Compatible?	174
12.2	Changing Coordinates in the Action	176
12.3	Adding Actions and Multiplying Probabilities	178
12.4	Rutherford Scattering	180
12.5	The Classical Version of Quantum Mechanics	184
12.6	The Propagator in Momentum Space	186
12.7	The Classical Green's Function	188
12.8	The Hydrogen Atom in Momentum Space	190
13.	The New World of Quantum Mechanics	194
13.1	The Liberation from Classical Chaos	194
13.2	The Time-Dependent Schrödinger Equation	196
13.3	The Stationary Schrödinger Equation	198
13.4	Feynman's Path Integral	200
13.5	Changing Coordinates in the Path Integral	202
13.6	The Classical Limit	204

14.	The Quantization of Integrable Systems	207
14.1	Einstein's Picture of Bohr's Quantization Rules	208
14.2	Keller's Construction of Wave Functions and Maslov Indices	211
14.3	Transformation to Normal Forms	215
14.4	The Frequency Analysis of a Classical Trajectory	220
14.5	The Adiabatic Principle	224
14.6	Tunneling Between Tori	227
15.	Wave Functions in Classically Chaotic Systems	231
15.1	The Eigenstates of an Integrable System	232
15.2	Patterns of Nodal Lines	233
15.3	Wave-Packet Dynamics	238
15.4	Wigner's Distribution Function in Phase Space	241
15.5	Correlation Lengths in Chaotic Wave Functions	247
15.6	Scars, or What Is Left of the Classical Periodic Orbits ...	249
16.	The Energy Spectrum of a Classically Chaotic System	254
16.1	The Spectrum as a Set of Numbers	255
16.2	The Density of States and Weyl's Formula	257
16.3	Measures for Spectral Fluctuations	261
16.4	The Spectrum of Random Matrices	263
16.5	The Density of States and Periodic Orbits	266
16.6	Level Clustering in the Regular Spectrum	270
16.7	The Fluctuations in the Irregular Spectrum	273
16.8	The Transition from the Regular to the Irregular Spectrum	275
16.9	Classical Chaos and Quantal Random Matrices	278
17.	The Trace Formula	282
17.1	The Van Vleck Formula Revisited	283
17.2	The Classical Green's Function in Action-Angle Variables	285
17.3	The Trace Formula for Integrable Systems	287
17.4	The Trace Formula in Chaotic Dynamical Systems	291
17.5	The Mathematical Foundations of the Trace Formula	295
17.6	Extensions and Applications	298
17.7	Sum Rules and Correlations	301
17.8	Homogeneous Hamiltonians	305
17.9	The Riemann Zeta-Function	307
17.10	Discrete Symmetries and the Anisotropic Kepler Problem	312
17.11	From Periodic Orbits to Code Words	314
17.12	Transfer Matrices	317

18.	The Diamagnetic Kepler Problem	322
18.1	The Hamiltonian in the Magnetic Field	323
18.2	Weak Magnetic Fields and the Third Integral	325
18.3	Strong Fields and Landau Levels	326
18.4	Scaling the Energy and the Magnetic Field	329
18.5	Calculation of the Oscillator Strengths	332
18.6	The Chaotic Spectrum in Terms of Closed Orbits	336
19.	Motion on a Surface of Constant Negative Curvature	340
19.1	Mechanics in a Riemannian Space	341
19.2	Poincaré's Model of Hyperbolic Geometry	345
19.3	The Construction of Polygons and Tilings	348
19.4	The Geodesics on a Double Torus	354
19.5	Selberg's Trace Formula	358
19.6	Computations on the Double Torus	363
19.7	Surfaces in Contact with the Outside World	369
19.8	Scattering on a Surface of Constant Negative Curvature	374
19.9	Chaos in Quantum-Mechanical Scattering	377
19.10	The Classical Interpretation of the Quantal Scattering ...	379
20.	Scattering Problems, Coding, and Multifractal Invariant Measures	383
20.1	Electron Scattering in a Muffin-Tin Potential	384
20.2	The Coding of Geodesics on a Singular Polygon	389
20.3	The Geometry of the Continued Fractions	393
20.4	A New Measure in Phase Space Based on the Coding ...	395
20.5	Invariant Multifractal Measures in Phase Space	398
20.6	Multifractals in the Anisotropic Kepler Problem	402
20.7	Bundling versus Pruning a Binary Tree	407
	References	410
	Index	427