

Contents

Preface	vii
Prerequisites	xv
CHAPTER 1	
Examples	1
1.1. Introduction	1
1.2. Iteration of Möbius Transformations	3
1.3. Iteration of $z \mapsto z^2$	6
1.4. Tchebychev Polynomials	9
1.5. Iteration of $z \mapsto z^2 - 1$	13
1.6. Iteration of $z \mapsto z^2 + c$	14
1.7. Iteration of $z \mapsto z + 1/z$	19
1.8. Iteration of $z \mapsto 2z - 1/z$	21
1.9. Newton's Approximation	22
1.10. General Remarks	25
CHAPTER 2	
Rational Maps	27
2.1. The Extended Complex Plane	27
2.2. Rational Maps	30
2.3. The Lipschitz Condition	32
2.4. Conjugacy	36
2.5. Valency	37
2.6. Fixed Points	38
2.7. Critical Points	43
2.8. A Topology on the Rational Functions	45

CHAPTER 3

The Fatou and Julia Sets	49
3.1. The Fatou and Julia Sets	49
3.2. Completely Invariant Sets	51
3.3. Normal Families and Equicontinuity	55
Appendix I. The Hyperbolic Metric	60

CHAPTER 4

Properties of the Julia Set	65
4.1. Exceptional Points	65
4.2. Properties of the Julia Set	67
4.3. Rational Maps with Empty Fatou Set	73
Appendix II. Elliptic Functions	77

CHAPTER 5

The Structure of the Fatou Set	80
5.1. The Topology of the Sphere	80
5.2. Completely Invariant Components of the Fatou Set	82
5.3. The Euler Characteristic	83
5.4. The Riemann–Hurwitz Formula for Covering Maps	85
5.5. Maps Between Components of the Fatou Set	90
5.6. The Number of Components of the Fatou Set	93
5.7. Components of the Julia Set	95

CHAPTER 6

Periodic Points	99
6.1. The Classification of Periodic Points	99
6.2. The Existence of Periodic Points	101
6.3. (Super)Attracting Cycles	104
6.4. Repelling Cycles	109
6.5. Rationally Indifferent Cycles	110
6.6. Irrationally Indifferent Cycles in F	132
6.7. Irrationally Indifferent Cycles in J	142
6.8. The Proof of the Existence of Periodic Points	145
6.9. The Julia Set and Periodic Points	148
6.10. Local Conjugacy	150
Appendix III. Infinite Products	155
Appendix IV. The Universal Covering Surface	157

CHAPTER 7

Forward Invariant Components	160
7.1. The Five Possibilities	160
7.2. Limit Functions	162
7.3. Parabolic Domains	165
7.4. Siegel Discs and Herman Rings	167
7.5. Connectivity of Invariant Components	172

CHAPTER 8

The No Wandering Domains Theorem 176

8.1. The No Wandering Domains Theorem 176

8.2. A Preliminary Result 177

8.3. Conformal Structures 179

8.4. Quasiconformal Conjugates of Rational Maps 183

8.5. Boundary Values of Conjugate Maps 184

8.6. The Proof of Theorem 8.1.2 186

CHAPTER 9

Critical Points 192

9.1. Introductory Remarks 192

9.2. The Normality of Inverse Maps 193

9.3. Critical Points and Periodic Domains 194

9.4. Applications 199

9.5. The Fatou Set of a Polynomial 202

9.6. The Number of Non-Repelling Cycles 210

9.7. Expanding Maps 223

9.8. Julia Sets as Cantor Sets 227

9.9. Julia Sets as Jordan Curves 232

9.10. The Mandelbrot Set 238

CHAPTER 10

Hausdorff Dimension 246

10.1. Hausdorff Dimension 246

10.2. Computing Dimensions 248

10.3. The Dimension of Julia Sets 251

CHAPTER 11

Examples 257

11.1. Smooth Julia Sets 257

11.2. Dendrites 258

11.3. Components of F of Infinite Connectivity 258

11.4. F with Infinitely Connected and Simply Connected Components 260

11.5. J with Infinitely Many Non-Degenerate Components 261

11.6. F of Infinite Connectivity with Critical Points in J 262

11.7. A Finitely Connected Component of F 263

11.8. J Is a Cantor Set of Circles 266

11.9. The Function $(z - 2)^2/z^2$ 271

References 273

Index of Examples 278

Index 279