

Contents

<i>Chapter I. The concept of stability and systems with constant coefficients.</i>	1
§ 1. Some remarks on the concept of stability	1
1.1. Existence, uniqueness, continuity	1
1.2. Stability in the sense of LYAPUNOV	4
1.3. Examples	6
1.4. Boundedness	7
1.5. Other types of requirements and comments	8
1.6. Stability of equilibrium	9
1.7. Variational systems	10
1.8. Orbital stability	12
1.9. Stability and change of coordinates	12
1.10. Stability of the m -th order in the sense of G. D. BIRKHOFF	13
1.11. A general remark and bibliographical notes	14
§ 2. Linear systems with constant coefficients	14
2.1. Matrix notations	14
2.2. First applications to differential systems	18
2.3. Systems with constant coefficients	19
2.4. The ROUTH-HURWITZ and other criteria	21
2.5. Systems of order 2	24
2.6. Nonhomogeneous systems	26
2.7. Linear resonance	27
2.8. Servomechanisms	28
2.9. Bibliographical notes	33
<i>Chapter II. General linear systems.</i>	34
§ 3. Linear systems with variable coefficients	34
3.1. A theorem of LYAPUNOV	34
3.2. A proof of (3.1.i).	35
3.3. Boundedness of the solutions	36
3.4. Further conditions for boundedness	37
3.5. The reduction to L -diagonal form and an outline of the proofs of (3.4. iii) and (3.4. iv)	39
3.6. Other conditions	41
3.7. Asymptotic behavior	41
3.8. Linear asymptotic equilibrium	42
3.9. Systems with variable coefficients	44
3.10. Matrix conditions	48
3.11. Nonhomogeneous systems	49
3.12. LYAPUNOV's type numbers	50
3.13. First application of type numbers to differential equations	51
3.14. Normal systems of solutions	52
3.15. Regular differential systems	53
3.16. A relation between type numbers and generalized characteristic roots	54
3.17. Bibliographical notes	55

§ 4. Linear systems with periodic coefficients	55
4.1. Floquet theory	55
4.2. Some important applications	59
4.3. Further results concerning equation (4.2.1) and extensions	61
4.4. Mathieu equation	65
4.5. Small periodic perturbations	66
4.6. Bibliographical notes	79
§ 5. The second order linear differential equation and generalizations	80
5.1. Oscillatory and non-oscillatory solutions	80
5.2. FUBINI's theorems	81
5.3. Some transformations	84
5.4. BELLMAN's and PRODI's theorems	84
5.5. The case $f(t) \rightarrow +\infty$	85
5.6. Solutions of class L^2	86
5.7. Parseval relation for functions of class L^2	88
5.8. Some properties of the spectrum S	89
5.9. Bibliographical notes	89
<i>Chapter III. Nonlinear systems</i>	91
§ 6. Some basic theorems on nonlinear systems and the first method of LYAPUNOV	91
6.1. General considerations	91
6.2. A theorem of existence and uniqueness	91
6.3. Periodic solutions of periodic systems	96
6.4. Periodic solutions of autonomous systems	98
6.5. A method of successive approximations and the first method of LYAPUNOV	99
6.6. Some results of BYLOV and VINOGRAD	101
6.7. The theorems of BELLMAN	102
6.8. Invariant measure	103
6.9. Differential equations on a torus	106
6.10. Bibliographical notes	107
§ 7. The second method of LYAPUNOV	107
7.1. The function V of LYAPUNOV	107
7.2. The theorems of LYAPUNOV	109
7.3. More recent results	111
7.4. A particular partial differential equation	113
7.5. Autonomous systems	114
7.6. Bibliographical notes	114
§ 8. Analytical methods	115
8.1. Introductory considerations	115
8.2. Method of LINDSTEDT	116
8.3. Method of POINCARÉ	118
8.4. Method of KRYLOV and BOGOLYUBOV, and of VAN DER POL	120
8.5. A convergent method for periodic solutions and existence theo- rems	123
8.6. The perturbation method	136
8.7. The Liénard equation and its periodic solutions	139
8.8. An oscillation theorem for the equation (8.7.1)	143
8.9. Existence of a periodic solution of equation (8.7.1)	145
8.10. Nonlinear free oscillations	145

8.11. Invariant surfaces	148
8.12. Bibliographical notes	150
8.13. Nonlinear resonance	150
8.14. Prime movers	151
8.15. Relaxation oscillation	155
§ 9. Analytical-topological methods	156
9.1. Poincaré theory of the critical points	156
9.2. Poincaré-Bendixson theory	163
9.3. Indices	167
9.4. A configuration concerning LIÉNARD's equation	170
9.5. Another existence theorem for the Liénard equation	174
9.6. The method of the fixed point	176
9.7. The method of M. L. CARTWRIGHT	177
9.8. The method of T. WAŻEWSKI	179
<i>Chapter IV. Asymptotic developments</i>	182
§ 10. Asymptotic developments in general	182
10.1. POINCARÉ's concept of asymptotic development	182
10.2. Ordinary, regular and irregular singular points	184
10.3. Asymptotic expansions for an irregular singular point of finite type	186
10.4. Asymptotic developments deduced from Taylor expansions	187
10.5. Equations containing a large parameter	189
10.6. Turning points and the theory of R. E. LANGER	192
10.7. Singular perturbation	195
<i>Bibliography</i>	197
<i>Index</i>	267