TABLE OF CONTENTS

Prefa	ce	xix
Intro	luction	xxi
	PART ONE. STABILITY OF CONTROL SYSTEMS	
Chapt	er I. Continuous and Discrete Deterministic Systems	3
§ 1.	Basic Definitions of Stability Theory for Continuous	
	systems	3
	1. Stability	3
	2. Asymptotic stability	8
	3. Other stability definitions	10
§ 2.	Lyapunov's Direct Method	11
§ 3.	Examples of the Use of Lyapunov's Method	16
	1. Stability of the motion of a shell	16
	2. Motion of a rigid body fastened at one point	18
§ 4.	Development of Lyapunov's Method	19
	 The Barbashin-Krasovskii theorem 	20
	2. The Matrosov criterion	22
	3. The comparison principle	23
	4. Stability with respect to part of the variables (partial	
	stability)	24
§ 5.	Stability of Linear Time-Invariant Systems	25
	1. The Routh-Hurwitz criterion	25
	2. Frequency criteria of stability	29
	3. Automatic frequency control system of a heterodyne	
	receiver	31
	4. Linear single-circuit automatic control systems	33
	5. Robust stability	34
§ 6.	Stability of Linear Time-Varying Equations	36
	1. On the method of "frozen" coefficients	36
	2. Systems with an almost constant matrix	37
	3. Linear systems with periodic coefficients	39
	4. Equation of the second order with periodic	4-
	coefficients	40
	5. Parametric resonance in engineering applications	42

§ 7.	Lyapunov Functions for Linear Time-Invariant Systems	
ŭ	and First Approximation Stability	43
	1. Lyapunov's matrix equation	43
	2. Stability in the first approximation	46
	3. Rotation of a shell	48
	4. Time-Varying equations of the first approximation	48
§ 8.	Synthesis of Control Systems for the Manipulator Robot	49
ŭ	1. Single-link manipulator robot	49
	2. The Cyclone-type robot	50
§ 9	Use of the Logarithmic Norm in Stability Theory	53
v	1. The definition and properties of the logarithmic	
	norm	53
	2. Stability of nonlinear systems	54
§ 10.	Use of Degenerate Lyapunov Functions	59
-	Stability of Discrete Systems	63
v	1. Lyapunov's direct method	64
	2. Linear time-invariant equations	65
	3. Stability in the first approximation	66
	4. Stability with respect to specified variables	66
	5. Unconditional minimization of functions	67
Main	Results and Formulas of Chapter I	69
Chap	ter II. Stability of Stochastic Systems	73
_	Introduction	73
•	Some Preliminaries from Probability Theory and the	
3 =-	Theory of Stochastic processes	73
	1. Basic probability space	73
	2. Random variables	74
	3. Stochastic processes	76
§ 3.	Stochastic Integrals and Stochastic Differential Equations	79
3 -	1. The stochastic integrals of Ito and Stratonovich	79
	2. The Ito formula	82
	3. Markov diffusion processes	86
	4. Linear stochastic equations	88
§ 4 .	Definition of Stochastic Stability	90
•	Application of Lyapunov's Direct Method	93
3	1. Sufficient stability conditions	93
	2. Stability in mean square of linear systems	96
	3. Scalar equations of the nth order	98
ξ 6.	Stability in Probability of Satellite Motion	99
J -	1. The pitch stability of a symmetric satellite in a	
	circular orbit	99
	2. The yaw angle stability of a satellite in a circular	
	equatorial orbit	100
Main	Results and Formulas of Chapter II	103

PART TWO. CONTROL OF DETERMINISTIC SYSTEMS

Chapter III. Description of Control Problems	127
§ 1. Introduction	127
§ 2. Statement of the Optimal Control Problem	128
1. The equations of evolution of a system	128
2. The functional to be minimized (cost functional)	129
3. Trajectory constraints	131
4. Control constraints	132
5. Joint constraints	135
§ 3. Examples of Optimal Control in Engineering	137
1. Optimal control of an electric motor	137
2. Optimization of the characteristics of nuclear	100
reactors	139
3. Optimal control of spacecraft	145
Main Results and Formulas of Chapter III	150
Chapter IV. The Classical Calculus of Variations and Optimal	
Control	151
§ 1. Problems with a Variable End Point and Fixed Time	151
1. Main assumptions	151
2. Cauchy's formula	152
3. Necessary conditions for optimal control	153
4. The Boltz problem	156
§ 2. Optimal Control of Linear Systems with a Quadratic	
Functional	157
1. Necessary conditions for optimal control	157
2. Construction of an optimal control	158
3. Matrix Riccati equation	159
4. The scalar case	163
Optimal control of wire reeling	166
§ 3. Necessary Conditions for Optimal Control. The Method	
of Lagrange Multipliers	168
1. The method of Lagrange multipliers	168
2. Fixed initial and terminal moments, and fixed initial	
state	172
3. Fixed initial and terminal moments, and variable	170
initial and terminal states	172
4. Problems with fixed values of some state variables at	170
the initial and terminal moments	173
5. Problems with an unspecified terminal moment	174 175
6. The Chaplygin problem	119

	7.	Maximization of the rocket velocity immediately before putting the rocket into a rectilinear	177
		trajectory	177
Main	Resu	ılts and Formulas of Chapter IV	180
Chapt	ter V	7. The Maximum Principle	183
§ 1.	Pro	blems with a Variable Terminal Point and Prescribed	
	Tra	nsfer Time	183
	1.	The Mayer problem	183
	2.	The Boltz problem	186
	3.	About solutions of maximum principle equations	188
	4.	Rotation of a motor shaft through a maximum angle	189
§ 2.	Pro	blems with an Unspecified Terminal Moment	190
	1.	Reduction to a Mayer problem	190
	2.	Necessary conditions for the optimality of systems	
		that are linear in a scalar control variable	191
	3.	Multivariable control	195
	4.	Time-invariant systems	196
	5 .	Transfer of a system from one manifold to another	
		manifold	196
	6.	Control problems with isoperimetric constraints	197
	7.	Sufficiency of the maximum principle	201
	8.	Connection between the maximum principle and the	
		classical calculus of variations	201
	9.	The maximum principle for discrete systems	202
§ 3.	Pra	ctical Applications of the Maximum Principle	205
	1.	Optimal configuration of a nuclear reactor	206
	2.	Control of a motion with regulated friction	208
	3.	Problem of the soft landing on the moon	214
	4.	Problem of the planar transfer of a space vehicle from	
		one circular orbit to another	217
§ 4.	Co	ntrol of Ecological Systems	219
	1.	Equations describing the evolution of a single	220
	0	population	222
	2.	Communities of at least two species	222
	3.	Statements of typical control problems for ecological	225
		systems The system of the "mandator may" gystem	22
	4.	Time-optimal control of the "predator-prey" system	22
Main	Res	ults and Formulas of Chapter V	234
Chap	ter ³	VI. Linear Control Systems	239
§ 1.	. A .	Fime-Optimal Problem	239
•	1.	Evaluation of the number of switch points	23
	2.	The damping of the material point	24
	2	The damning of a pendulum	24

	4.	Control of the rotation of an axially symmetric space	
		vehicle	246
	5.	The controllability set	249
ξ 2.	Con	trollability of Linear Systems	252
3	1.	Controllability of linear time-invariant systems	252
	2.	Controllability of linear time-varying systems	255
	3.	Canonical form of linear time-invariant control	
		systems	258
	4.	Canonical form of linear time-varying control	
		systems	260
	5 .	The Hautus criterion for controllability	261
	6.	Controllability of a two-link manipulator	263
8.3	Obs	servation in Linear Systems. Observers	265
3 0.	1.	Statement of an observation problem. Duality of	
		control and observation problems	265
	2.	On a method of determining the state vector	271
	3.	Observer of maximum order	272
	4.	Observer of reduced order (the Luenberger observer)	273
	5.	Observer of reduced order in the stabilization system	
	•	of an aircraft	276
8 4	Lin	ear Feedback Control Systems	280
3 -	1.	Various ways of describing feedback control systems.	
		The realization problem	280
	2.	Criteria of performance for SISO-systems	283
	3.	Criteria of performance for MIMO-systems. The	
	Ο.	Hardy spaces H_2 and H_{∞}	284
c =	т	and amentals of H_{∞} -Theory	286
3 0		Statement of the problem of constructing an H_{∞} -	
	1.	optimal controller	286
	2	Estimates of the H_{∞} and H_2 norms of the transfer	
	2.	matrix of an auxiliary system	287
	2	The H_{∞} -problem for a static controller	288
	3. 4.	The general case of a dynamic controller	290
	4. 5.	Robust stability	291
		ros of a Linear Time-Invariant System and Their Use	292
3 6	o. Ze	ros of a Linear Time-invariant System and During	
		sults and Formulas of Chapter VI	297
Cha	pter	VII. Dynamic Programming Approach. Sufficient	001
	\mathbf{C}	onditions for Optimal Control	301
§ 1	ı. Tł	ne Bellman Equation and its Properties	301
J -	1.	The principle of dynamic programming. Heuristic	
	- "	derivation of The Bellman equation	301
	2.	Determining an F-control with the help of the	
		dynamic programming approach	303

	3. Connection between the dynamic programming	
	approach and the maximum principle	305
	4. Determining F-control in the problem of damping	
	the motion of a rigid body	306
	5. Optimal procedure for reducing the power of a	
	nuclear reactor	307
	6. The linear-quadratic problem	308
8 2.	Control on an Unbounded Time Interval. Stabilization of	
3	Dynamical Systems	309
	1. Problem statements	309
	2. Lyapunov's direct method for the optimal	
	stabilization Problem	310
	3. Exponential stabilization	312
	4. Stabilization of the motion of a manipulator robot	315
8.3	Stabilization of Linear Systems	320
3 0.	1. Time-varying linear-quadratic problems	320
	2. The use of the method of successive approximations	020
	for determining optimal control	321
	3. Time-invariant linear-quadratic equation	322
	4. The algebraic Riccati equation	323
٤ ،	Stabilization of Quasilinear Systems	325
g 4.	1. Quasioptimal stabilization and estimation of its error	325
	2. Adaptive stabilization	331
6 =	•	001
g 5.	Sufficient Conditions for Optimal Control Using Auxiliary Functions	332
		332
	 Conditions for optimal control Sufficient conditions for the existence of a minimizing 	332
		337
	sequence 3. Problems with unspecified time	337
	<u> </u>	
Main	Results and Formulas of Chapter VII	339
Chan	ter VIII. Some Additional Topics of Optimal Control	
Chap	Theory	343
8 1	Existence of Optimal Control	343
3 1.	1. Problem statement and the main assumptions	343
	2. Main theorem	34
	3. Analysis of the conditions of the main theorem	349
6.2	·	35
g 2.	Singular Optimal Control	35
	 The definition and determination of singular controls Optimality of singular control 	35.
	1 0	30
	3. Generalization of the Kelley and Kopp-Moyer	25
	conditions	35
-	Chattering Mode	35
§ 4.	Sliding Optimal Mode	36
Main	Results and Formulas of Chapter VIII	37

PART THREE. OPTIMAL CONTROL OF DYNAMICAL SYSTEMS UNDER RANDOM DISTURBANCES

Chapter IX. Control of Stochastic Systems. Problem	
Statements and Investigation Techniques	393
§ 1. Statements of Control Problems for Stochastic Systems	393
1. Equations of motion of a system	393
2. Control constraints	394
3. Cost functional	396
§ 2. Dynamic Programming Approach	39 8
1. The Bellman function	398
2. The Bellman equation	399
3. Connection between the Bellman function and the	
Bellman equation	403
§ 3. Stochastic Linear-Quadratic Problem on a Finite Time	
Interval	406
1. Linear-quadratic problems in the case of an accurate	400
measurement of phase coordinates	406
2. Linear-quadratic problems under incomplete	409
information 3 Optimal program control in linear-quadratic	409
o. Optimen programm	412
problems 4. Linear-quadratic problem for Gaussian and Poisson	412
4. Linear-quadratic problem for Gaussian and Poisson disturbances	415
and to rendom	
5. Control of wire reeling with regard to random disturbances	416
§ 4. Control on an Unbounded Time Interval. Stabilization of	
Stochastic Control Systems	417
1. Problem statement	417
2. Application of Lyapunov's direct method to optimal	
stabilization problems	417
3 Stabilization of linear stochastic systems	421
§ 5 Approximate Methods for Determining Optimal Control	422
1. Description of the algorithm of successive	
approximations	423
2. Zero approximation estimate	424
3 First approximation estimate	427
4. Higher-order approximation estimates	429
Main Results and Formulas of Chapter IX	431
Chapter X. Optimal Control on a Time Interval of Random	
Duration	435
§ 1. Time-Optimal Control	435
§ 1. Time-Optimal Control	

	1.	Statement of time-optimal problems in dynamical	
		systems under random disturbances	435
	2.	Existence of an admissible control	436
	3.	An algorithm for constructing optimal control	438
	4.	Time-optimal control of the motion of a rigid body	439
	5.	Numerical construction of the time-optimal control	
		of the motion of a material point	441
§ 2.	Tim	ne-Optimality for a Gyrostat	444
Ū	1.	Problem statement	444
	2.	The existence of an admissible control	446
	3.	Construction of an optimal control	448
ξ3.	Con	ntrol Problems with Stochastic Functional	450
•	1.	Problem statement and method of solution	450
	2.	Optimal control of the motion of a material point	
		involving a stochastic cost functional	452
	3.	Maximization of the probability that a point stays in	
	-	a given region	455
	4.	Stochastic control of the motion of a simple	
		pendulum	45
	5 .	Control of a rigid body, given the stochastic cost	
		functional	460
	6.	Maximization of the mean time for the system	
	-	staying within a given region. Determination of	
		optimal control for the motion of a rigid body	46
Main	Post	ults and Formulas of Chapter X	460
		KI. Optimal Estimation of the State of the System	46'
§ 1.		imation Problems Involving Random Disturbances	46
	1.	Statement of an optimal estimation problem	46
	2.	Linear estimation	46
	3.	Optimal estimation of Gaussian variables	469
	4.	Linear estimation of stationary processes. The	10
		Wiener-Hopf equation	46
§ 2.		e Kalman filter	47
	1.	Problem statement	47
	2.	The dual problem of optimal control	47
	3.	Equation for the estimation error	47
	4.	Equation for the optimal estimate	47
	5 .	Stability of the filter	47
	6.	Filtering problem with constant parameters	48
	7.	Filtering problem with degenerate noise in the	
		observation channel	48
	8.	Optimal extrapolation	48
	9.	Optimal interpolation	48
	10.	The discrete Kalman filter	48
83	Sor	ne Relations of Nonlinear Filtering Theory	48

1.	Statement of the general filtering problem	488
2.	The Stratonovich differential equations for the	
	conditional probability distribution	489
3.	Conditionally Gaussian processes	490
4.	Quasioptimal and quasilinear filtering (the scalar	
	case)	491
5.	Quasilinear filtering (the multidimensional case)	499
Main Res	ults and Formulas of Chapter XI	503
Chapter 2	II. Optimal Control of the Observation Process	509
	timization of the Observation Process	509
1.	Problem statement. Basic relations	509
2.	Construction of optimal observation laws minimizing	
	terminal variance	513
3.	An example of observation control with integral cost	
_	functional	515
4.	Optimal pulse observation laws	517
5.	Pulsed observation of a material point	522
6.	Optimal noise control in the observation channel	524
	timal Combination of Control and Observation	529
3 2. Op	Linear-quadratic problem	529
2.	A deterministic scalar system	532
3.	A stochastic scalar system	534
Main Res	sults and Formulas of Chapter XII	542
Exercises	of Part Three	543
Par	FOUR. NUMERICAL METHODS IN CONTROL SYSTEM	S
Chapter	XIII. Linear Time-Invariant Control Systems	555
8 1 St:	ability of Linear Time-Invariant Systems	555
3 1. 50	Precise methods for solving the complete eigenvalue	
	problem	555
2.	Iterative methods	559
3.	The Routh-Hurwitz criterion	561
4.	The Zubov method of a functional transformation of	
-1.	a matrix	562
S O M	ethods of Solving the Lyapunov Equation	563
3 2. M	Preliminary remarks	563
2.	The series method	564
2. 3.	Method of the matrix sign function	565
3. 4.	Application of the QR-algorithm	567
4. 5.	Construction of stabilizing control	568
5. 6.	Computation of the covariance matrix	570
	ontrollability and Observability	570
3 3. Co	office of the contraction of the	

§ 4.	Linear-Quadratic Time-Invariant Problem of Optimal stabilization	572
	1. Reduction to a sequence of Lyapunov equations	572
	2. Use of the QR-algorithm	573
	3. Landing control of a Boeing-747 airplane	576
Main :	Results and Formulas of Chapter XIII	579
Chapt	er XIV. Numerical Methods for the Investigation of	F01
	Nonlinear Control Systems	581
§ 1.	Analysis of Transients. The Runge-Kutta Methods	581
	1. On numerical methods of investigating systems	581
	2. One-step methods	581 585
	 Error estimates for Runge-Kutta methods Estimation of solution errors for stable equations 	588
	5. Standard programs	589
6.3	Analysis of Transients. Multistep Methods	591
3 2.	1. General definitions	591
	2. Particular multistep formulas	591
	3. Computational scheme for multistep formulas	594
	4. Error estimation	596
	5. The Butcher formulas	599
§ 3.	Stiff Systems of Equations	600
§ 4.	Numerical Methods for the Design of Optimal Control in	
	the Linear-Quadratic Problem	607
	1. Determination of an optimal F-control for time-	401
	varying systems	607
	2. Time-invariant linear-quadratic problem on a finite interval	608
	3. Optimal stabilization problems and methods for	
	solving the algebraic Riccati equation	612
	4. The numerical design of the Kalman filter	613
Main	Results and Formulas of Chapter XIV	617
Chapt	ter XV. Numerical Design of Optimal Control Systems	619
	On Numerical Methods of Solving Optimal Control	
3	Problems	619
8 2	Reduction to Nonlinear Programming	620
	Reduction to a Boundary-Value Problem	62
3 U .	1. Boundary-value problem of the maximum principle	62
	2. The Newton method of solving a boundary-value	
	problem	623
	3. An example of reducing an optimal control problem	
	to a boundary-value problem	623
2.4	Solution of Linear Roundary-Value Problems	62

1. Reduction of an optimal control problem to a linear	
boundary-value problem 6	25
2. Reduction of a linear boundary-value problem to	
Cauchy problems	26
J. Method of the transfer of boundary constraints	28
4. The Holamov memod	3 0
§ 5. The Shatrovskii Method of Successive Improvements of	
Control	32
§ 6. The Fedorenko Method of Reducing to a Linear	
Programming Problem 6	35
Main Results and Formulas of Chapter XV	39
Exercises of Part Four	41
General References	57
Subject Index 6	63