Contents

1 Introduction 1

1.1 A Lyapunov framework for robust control 3
1.2 Inverse optimality in robust stabilization 6
1.3 Recursive Lyapunov design 9

2 Set-Valued Maps 15

2.1 Continuity of set-valued maps 17
2.1.1 Upper and lower semicontinuity 17
2.1.2 Lipschitz and Hausdorff continuity 19
2.2 Marginal functions 21
2.3 Intersections 23
2.3.1 Continuity of intersections 23
2.3.2 Lipschitz continuity of intersections 24
2.4 Selection theorems 28
2.4.1 Michael's theorem 28
2.4.2 Minimal selections 28
2.4.3 Lipschitz selections 29
2.5 Parameterized maps 30
2.6 Summary 32

3 Robust Control Lyapunov Functions 33

3.1 Nonlinear robust stabilization 35
3.1.1 System description 35
3.1.2 Problem statement 39
3.2 Nonlinear disturbance attenuation 40
3.2.1 Input-to-state stability 41
3.2.2 Nonlinear small gain theorems 42
3.2.3 Disturbance attenuation vs. robust stabilization . . 43
3.3 Robust control Lyapunov functions 45
3.3.1 Control Lyapunov functions 46
3.3.2 Rclf: general definition 48
3.3.3 Rclf: state-feedback for time-invariant systems . . . 49

3.3.4 Rclf: absence of disturbance input 51
3.4 Rclf implies robust stabilizability 53
3.4.1 Small control property 56
3.4.2 Output feedback 58
3.4.3 Locally Lipschitz state feedback 60
3.5 Robust stabilizability implies rclf 61
3.6 Summary 63

4 Inverse Optimality **65**
4.1 Optimal stabilization: obstacles and benefits 66
4.1.1 Inverse optimality, sensitivity reduction, and stability margins 67
4.1.2 An introductory example 69
4.2 Pointwise min-norm control laws 71
4.2.1 General formula 72
4.2.2 Jointly affine systems 75
4.2.3 Feedback linearizable systems 76
4.3 Inverse optimal robust stabilization 78
4.3.1 A preliminary result 78
4.3.2 A differential game formulation 79
4.3.3 Main theorem 81
4.4 Proof of the main theorem 83
4.4.1 Terminology and technical lemmas 83
4.4.2 Construction of the function r 85
4.4.3 Proof of the key proposition 88
4.4.4 Proof of optimality 91
4.5 Extension to finite horizon games 93
4.5.1 A finite horizon differential game 94
4.5.2 Main theorem: finite horizon 95
4.5.3 Proof of the main theorem 96
4.6 Summary 100

5 Robust Backstepping **101**
5.1 Lyapunov redesign 103
5.1.1 Matched uncertainty 103
5.1.2 Beyond the matching condition 105
5.2 Recursive Lyapunov design 107
5.2.1 Class of systems: strict feedback form 108
5.2.2 Construction of an rclf 110
5.2.3 Backstepping design procedure 115
5.2.4 A benchmark example 117
5.3 Flattened rclf's for softer control laws 119

5.3.1 Hardening of control laws . . . 119
5.3.2 Flattened rclf's . . . 123
5.3.3 Design example: elimination of chattering . . . 126
5.4 Nonsmooth backstepping . . . 127
5.4.1 Clarke's generalized directional derivative . . . 130
5.4.2 Nonsmooth rclf's . . . 131
5.4.3 Backstepping with nonsmooth nonlinearities . . . 132
5.5 Summary . . . 136

6 Measurement Disturbances **137**
6.1 Effects of measurement disturbances . . . 138
6.1.1 Loss of global stability . . . 138
6.1.2 Loss of global stabilizability . . . 139
6.2 Design for strict feedback systems . . . 143
6.2.1 Measurement constraint for ISS . . . 143
6.2.2 Backstepping with measurement disturbances . . . 145
6.2.3 Initialization step . . . 148
6.2.4 Recursion step . . . 150
6.2.5 Design procedure and example . . . 157
6.3 Summary . . . 160

7 Dynamic Partial State Feedback **161**
7.1 Nonlinear observer paradigm . . . 162
7.1.1 Extended strict feedback systems . . . 162
7.1.2 Assumptions and system structure . . . 163
7.2 Controller design . . . 167
7.2.1 Main result . . . 167
7.2.2 Controller design for $n = 1$ . . . 168
7.2.3 Conceptual controllers and derivatives . . . 172
7.2.4 Backstepping lemma . . . 174
7.2.5 Controller design for $n \geq 2$ . . . 177
7.2.6 Proof of the main result . . . 179
7.3 Design example . . . 180
7.3.1 Truth model and design model . . . 182
7.3.2 Full state feedback design . . . 186
7.3.3 Partial state feedback design . . . 194
7.4 Summary . . . 201

8 Robust Nonlinear PI Control **203**
8.1 Problem formulation . . . 204
8.1.1 Class of systems . . . 204
8.1.2 Design objective . . . 206
8.2 Controller design . . . 208

8.2.1 Main result 208
8.2.2 Technical lemma 209
8.2.3 Controller design for $r = 1$ 211
8.2.4 Backstepping construction 215
8.2.5 Controller design for $r \geq 2$ 218
8.2.6 Proof of the main result 222
8.3 Design example 223
8.4 Summary 227

Appendix: Local $\mathcal{K}$-continuity in metric spaces **229**
A.1 $\mathcal{K}$-continuity 230
A.2 Local $\mathcal{K}$-continuity 233
A.3 $C\mathcal{K}$-continuity 237

Bibliography **241**

Index **255**