

Contents

Chapter 1. Max and Min	1
A. Penalties and Constraints	2
B. Epigraphs and Semicontinuity	7
C. Attainment of a Minimum	11
D. Continuity, Closure and Growth	13
E. Extended Arithmetic	15
F. Parametric Dependence	16
G. Moreau Envelopes	20
H. Epi-Addition and Epi-Multiplication	23
I* Auxiliary Facts and Principles	28
Commentary	34
Chapter 2. Convexity	38
A. Convex Sets and Functions	38
B. Level Sets and Intersections	42
C. Derivative Tests	45
D. Convexity in Operations	49
E. Convex Hulls	53
F. Closures and Continuity	57
G* Separation	62
H* Relative Interiors	64
I* Piecewise Linear Functions	67
J* Other Examples	71
Commentary	74
Chapter 3. Cones and Cosmic Closure	77
A. Direction Points	77
B. Horizon Cones	80
C. Horizon Functions	87
D. Coercivity Properties	91
E* Cones and Orderings	96
F* Cosmic Convexity	97

G* Positive Hulls	100
Commentary	105
Chapter 4. Set Convergence	108
A. Inner and Outer Limits	109
B. Painlevé-Kuratowski Convergence	111
C. Pompeiu-Hausdorff Distance	117
D. Cones and Convex Sets	118
E. Compactness Properties	120
F. Horizon Limits	122
G* Continuity of Operations	125
H* Quantification of Convergence	131
I* Hyperspace Metrics	138
Commentary	144
Chapter 5. Set-Valued Mappings	148
A. Domains, Ranges and Inverses	149
B. Continuity and Semicontinuity	152
C. Local Boundedness	157
D. Total Continuity	164
E. Pointwise and Graphical Convergence	166
F. Equicontinuity of Sequences	173
G. Continuous and Uniform Convergence	175
H* Metric Descriptions of Convergence	181
I* Operations on Mappings	183
J* Generic Continuity and Selections	187
Commentary	192
Chapter 6. Variational Geometry	196
A. Tangent Cones	196
B. Normal Cones and Clarke Regularity	199
C. Smooth Manifolds and Convex Sets	202
D. Optimality and Lagrange Multipliers	205
E. Proximal Normals and Polarity	212
F. Tangent-Normal Relations	217
G* Recession Properties	222
H* Irregularity and Convexification	225
I* Other Formulas	227
Commentary	232
Chapter 7. Epigraphical Limits	238
A. Pointwise Convergence	239
B. Epi-Convergence	240
C. Continuous and Uniform Convergence	250
D. Generalized Differentiability	255

E.	Convergence in Minimization	262
F.	Epi-Continuity of Function-Valued Mappings	270
G.*	Continuity of Operations	275
H.*	Total Epi-Convergence	278
I.*	Epi-Distances	282
J.*	Solution Estimates	286
	Commentary	292
Chapter 8.	Subderivatives and Subgradients	298
A.	Subderivatives of Functions	299
B.	Subgradients of Functions	300
C.	Convexity and Optimality	308
D.	Regular Subderivatives	311
E.	Support Functions and Subdifferential Duality	317
F.	Calmmess	322
G.	Graphical Differentiation of Mappings	324
H.*	Proto-Differentiability and Graphical Regularity	329
I.*	Proximal Subgradients	333
J.*	Other Results	336
	Commentary	343
Chapter 9.	Lipschitzian Properties	349
A.	Single-Valued Mappings	349
B.	Estimates of the Lipschitz Modulus	354
C.	Subdifferential Characterizations	358
D.	Derivative Mappings and Their Norms	364
E.	Lipschitzian Concepts for Set-Valued Mappings	368
F.	Aubin Property and Mordukhovich Criterion	376
G.	Metric Regularity and Openness	386
H.*	Semiderivatives and Strict Graphical Derivatives	390
I.*	Other Properties	399
J.*	Rademacher's Theorem and Consequences	403
K.*	Mollifiers and Extremals	408
	Commentary	415
Chapter 10.	Subdifferential Calculus	421
A.	Optimality and Normals to Level Sets	421
B.	Basic Chain Rule and Consequences	426
C.	Parametric Optimality	432
D.	Rescaling	438
E.	Piecewise Linear-Quadratic Functions	440
F.	Amenable Sets and Functions	442
G.	Semiderivatives and Subsmoothness	446
H.*	Coderivative Calculus	452
I.*	Extensions	458
	Commentary	469

Chapter 11. Dualization	473
A. Legendre–Fenchel Transform	473
B. Special Cases of Conjugacy	476
C. The Role of Differentiability	480
D. Piecewise Linear-Quadratic Functions	484
E. Polar Sets and Gauges	490
F. Dual Operations	493
G. Duality in Convergence	499
H. Dual Problems of Optimization	502
I. Lagrangian Functions	508
J* Minimax Problems	514
K* Augmented Lagrangians and Nonconvex Duality	518
L* Generalized Conjugacy	525
Commentary	529
Chapter 12. Monotone Mappings	533
A. Monotonicity Tests and Maximality	533
B. Minty Parameterization	537
C. Connections with Convex Functions	542
D. Graphical Convergence	551
E. Domains and Ranges	553
F* Preservation of Maximality	556
G* Monotone Variational Inequalities	558
H* Strong Monotonicity and Strong Convexity	562
I* Continuity and Differentiability	567
Commentary	575
Chapter 13. Second-Order Theory	579
A. Second-Order Differentiability	579
B. Second Subderivatives	582
C. Calculus Rules	591
D. Convex Functions and Duality	603
E. Second-Order Optimality	606
F. Prox-Regularity	609
G. Subgradient Proto-Differentiability	618
H. Subgradient Coderivatives and Perturbation	622
I* Further Derivative Properties	625
J* Parabolic Subderivatives	633
Commentary	638
Chapter 14. Measurability	642
A. Measurable Mappings and Selections	643
B. Preservation of Measurability	651
C. Limit Operations	655
D. Normal Integrands	660
E. Operations on Integrands	669

F. Integral Functionals	675
Commentary	679
References	684
Index of Statements	710
Index of Notation	725
Index of Topics	726