

Contents

Preface	xiii
Structure of the book and main contributions	xv
Acknowledgments	xviii
Acronyms, Notation, and Symbols	xix
I General Matrix Theory and Linear Dynamical Systems	1
1 Matrices, Matrix Pencils, and Rational Matrix Functions	3
1.1 The eigenvalue problem	3
1.2 The Jordan canonical form of a square matrix	4
1.3 Invariant subspace of a square matrix	5
1.4 Controllability and observability	7
1.5 Linear matrix equations	9
1.6 The generalized eigenvalue problem: the regular case	12
1.7 The Weierstrass canonical form of a regular matrix pencil	13
1.8 Deflating subspace of a regular matrix pencil	14
1.9 The generalized eigenvalue problem: the general case	16
1.10 The Kronecker canonical form of a general matrix pencil	17
1.11 Proper deflating subspace of a general matrix pencil	18
1.12 Structural elements of a rational matrix	22
1.13 Realization theory for rational matrices	24
1.13.1 The proper case	24
1.13.2 The general case	26
1.14 Structural elements in terms of realizations for a rational matrix	28
Notes and references	30
2 Linear Dynamical Systems	31
A. The continuous-time case	31
2.1 Linear systems	31
2.1.1 Stable, antistable, and dichotomic systems	32
2.1.2 Controllability, stabilizability, and duals	33
2.1.3 Spaces of rational matrices, norms, and continuity	33
2.1.4 Gramians and Hankel singular values	36

2.1.5	Connections of systems	37
2.2	State evolutions corresponding to L^2 inputs and associated operators	42
2.2.1	The case: A stable	42
2.2.2	The case: A antistable	44
2.2.3	The case: A dichotomic	45
2.2.4	The case: A arbitrary	45
2.3	The L^2 input-output operator	46
2.4	Hankel and Toeplitz operators	47
B.	The discrete-time case	52
2.5	Linear discrete-time systems	52
2.5.1	Direct and inverse-time systems	52
2.5.2	Stable, antistable, and dichotomic systems	54
2.5.3	Controllability, stabilizability, and duals	55
2.5.4	Spaces of rational matrices, norms, and continuity	55
2.5.5	Gramians and Hankel singular values	57
2.5.6	Bilinear transformation between a continuous and a discrete-time system	58
2.5.7	Continuity of the H^∞ -norm	59
2.6	State evolutions corresponding to ℓ^2 inputs and associated operators	62
2.6.1	The direct-time case: A stable	63
2.6.2	The inverse-time case: A stable	65
2.6.3	The direct-time case: A antistable	65
2.6.4	The direct-time case: A dichotomic	66
2.6.5	The inverse-time case: A dichotomic	67
2.6.6	The case: A arbitrary	68
2.7	ℓ^2 input-output operators	68
2.7.1	The direct-time case	68
2.7.2	The inverse-time case	69
2.8	Hankel and Toeplitz operators	70
	Notes and references	74

II Generalized Riccati Theory 75

3	Popov Triplets	77
A.	The continuous-time case	77
3.1	Popov triplets: definition, significance, and equivalence	77
3.2	Objects associated with a Popov triplet	79
3.2.1	Quadratic indices	79
3.2.2	The continuous-time algebraic Riccati system (CTARS)	80
3.2.3	The continuous-time algebraic Riccati equation (CTARE)	82
3.2.4	The Kalman-Popov-Yakubovich system (KPYS)	83
3.2.5	The continuous-time Hamiltonian system (CTHS)	85
3.2.6	The extended Hamiltonian pencil (EHP)	86
3.2.7	The continuous-time Popov function	86
3.3	Associated operators	88
3.3.1	The case: A dichotomic	88

3.3.2	The case: A stable	89
3.3.3	The case: A antistable	91
3.3.4	Reduction of the case A antistable to the case A stable	92
3.3.5	\mathcal{R}_e as the input–output operator of the CTHS	93
3.4	Particular relevant cases	94
B.	The discrete–time case	95
3.5	Popov triplets: the double interpretation, significance, and equivalence	95
3.6	Objects associated with a Popov triplet in discrete–time	97
3.6.1	Quadratic indices	97
3.6.2	The discrete–time algebraic Riccati system (DTARS)	98
3.6.3	The discrete–time algebraic Riccati equation (DTARE)	99
3.6.4	The Kalman–Szegő–Popov–Yakubovich system (KSPYS)	100
3.6.5	The discrete–time Hamiltonian system (DTHS)	102
3.6.6	The extended symplectic pencils (ESPs)	103
3.6.7	The discrete–time Popov functions	104
3.7	The time–reversed Popov triplet	105
3.8	Associated operators	108
3.8.1	The direct–time case: A dichotomic	108
3.8.2	The direct–time case: A stable	109
3.8.3	The inverse–time case: A dichotomic	110
3.8.4	The inverse–time case: A stable	111
3.8.5	Reduction of the inverse–time case to the direct–time case	112
3.8.6	The direct–time case: A antistable	114
3.8.7	\mathcal{R}_e and $\widehat{\mathcal{R}}_e$ as input–output operators	115
	Notes and references	116
4	Riccati Theory: An Operator–based Approach	117
A.	The continuous–time case	118
4.1	Existence of the stabilizing solution: the stable case	118
4.1.1	The main result	118
4.1.2	Proof of the implication 2. \Rightarrow 1.	119
4.1.3	The positiveness case	121
4.2	Removing the stability assumption	123
4.2.1	The positiveness case: the standard Riccati equation for control	124
4.3	Existence of the stabilizing solution: the antistable case	125
4.3.1	The main result	125
4.3.2	The positiveness case	127
4.4	The signature condition	128
4.5	Frequency domain conditions	132
4.5.1	The main result	132
4.5.2	The positiveness case: A stable	134
4.5.3	The positiveness case: A arbitrary	135
4.5.4	The signature condition in the frequency domain	135
4.6	Algebraic Riccati inequalities	144
B.	The discrete–time case	145
4.7	Existence of the stabilizing solution: the stable case	145
4.7.1	The main result	146

4.7.2	Proof of the implication $2. \Rightarrow 1.$	147
4.7.3	The positiveness case	150
4.8	Removing the stability assumption	152
4.9	Existence of the stabilizing solution: the inverse-time case	152
4.10	Existence of the stabilizing solution: the antistable case	152
4.10.1	The main result	153
4.10.2	The positiveness case	155
4.11	The signature condition	156
4.12	Frequency domain conditions	157
4.12.1	The positiveness case: A arbitrary	159
4.12.2	The signature condition in the frequency domain	159
4.13	Algebraic Riccati inequalities	161
	Notes and references	162
5	Riccati Equations and Matrix Pencils: the Regular Case	163
A.	The continuous-time case	163
5.1	The eigenstructure of a regular EHP	163
5.2	The CTARE and the EHP	166
5.3	The continuous-time Bernoulli equation	169
B.	The discrete-time case	171
5.4	The eigenstructure of a regular ESP	171
5.5	The DTARE and the ESP	174
5.6	The discrete-time Bernoulli equation	177
5.7	The time-reversed DTARE	179
	Notes and references	185
6	Riccati Systems and Matrix Pencils: the General Case	187
A.	The continuous-time case	187
6.1	The eigenstructure of a general EHP	187
6.2	The generalized continuous-time Riccati system and the EHP	191
B.	The discrete-time case	194
6.3	The eigenstructure of a singular ESP	194
6.4	The generalized discrete-time Riccati system and the ESP	201
6.5	Numerical algorithms	204
6.5.1	Basic algorithms	204
6.5.2	Computation of proper deflating subspaces	209
6.5.3	Computation of solutions to Riccati equations and systems	212
	Notes and references	214
III	Applications to Systems Theory and Robust Control	215
7	Applications to Systems Theory	217
A.	The continuous-time case	217
7.1	The bounded real lemma	217
7.2	The CTARE associated with the positiveness Popov triplet	219
7.3	Normalized coprime factorizations	224

7.4	The Small Gain Theorem	225
7.5	A quadratic index with constrained dynamics	226
7.6	Spectral and inner–outer factorizations: the arbitrary rank case	228
	B. The discrete–time case	231
7.7	The bounded real lemma	231
7.8	The DTARE associated with the positiveness Popov triplet	232
7.9	Normalized coprime factorizations	233
7.10	The Small Gain Theorem	234
7.11	A quadratic index with constrained dynamics	235
7.12	Spectral and inner–outer factorization with respect to the unit circle	237
	Notes and references	240
8	The Four Block Nehari Problem	241
	A. The continuous–time case	241
8.1	The Nehari problem and the signature condition	241
8.2	Parrott’s problem	245
8.3	Necessary and sufficient conditions	247
8.4	Uncoupled solvability conditions	254
	B. The discrete–time case	257
8.5	The four block Nehari problem	257
	Notes and references	262
9	The Optimal H^2–Control Problem	263
	A. The continuous–time case	263
9.1	Problem formulation	263
9.1.1	An evaluation of the H^2 –norm	265
9.2	Main result	265
	B. The discrete–time case	271
9.3	Problem formulation	271
9.3.1	An evaluation of the H^2 –norm	271
9.4	Main result	272
	Notes and references	274
10	The H^∞–Control Problem	275
	A. The continuous–time case	275
10.1	Problem formulation	275
10.2	Basic assumptions	278
10.3	The solution	279
10.3.1	The solution under normalizing conditions	281
10.3.2	The plan of the proof	283
10.4	Redheffer’s theorem	284
10.5	Necessity of (C1) and (C2)	287
10.6	An auxiliary necessary condition	289
10.7	Necessity of (C3)	293
10.8	Proof of sufficiency	296
10.8.1	The disturbance estimation (DE) case	297
10.8.2	The disturbance feedforward (DF) case	300

10.8.3	The output estimation (OE) case	301
10.8.4	The general case	302
10.9	State feedback solution	302
B.	The discrete-time case	305
10.10	Problem formulation	305
10.11	Basic assumptions	306
10.12	The solution	307
10.13	Redheffer's theorem	309
10.14	Necessity of (CD1) and (CD2)	313
10.15	An auxiliary necessary condition	313
10.16	Necessity of (CD3)	315
10.17	Proof of sufficiency	318
10.17.1	The disturbance estimation (DE) case	318
10.17.2	The disturbance feedforward (DF) case	319
10.17.3	The output estimation (OE) case	320
10.17.4	The general case	320
10.18	State feedback solution	326
	Notes and References	328
11	Robust Stabilization	329
A.	The continuous-time case	329
11.1	Problem formulation and some prerequisites	329
11.2	Optimal solution to the DF problem	332
11.2.1	An evaluation of γ_{\min}	333
11.2.2	An optimal solution	336
11.3	Robust stabilization for normalized coprime factors uncertainties	343
11.3.1	An evaluation of the maximum stability margin	344
11.3.2	An optimal robustly stabilizing controller	346
11.4	Robust stabilization for multiplicative uncertainties	349
B.	The discrete-time case	352
11.5	Problem formulation and some prerequisites	352
11.6	Optimal solution to the DF problem	352
11.6.1	An evaluation of γ_{\min}	353
11.6.2	An optimal solution	353
11.7	Robust stabilization for normalized coprime factors uncertainties	359
11.7.1	An evaluation of the maximum stability margin	360
11.7.2	An optimal robustly stabilizing controller	363
11.8	Robust stabilization for multiplicative uncertainties	366
	Notes and references	368
References	369
Index	375