

## CONTENTS

Series Editor's Preface	v
Preface (to the Russian Edition)	vii
<b>Chapter 1</b> Basic concepts and statement of problems in control theory	1
1.1 Initial premises	1
1.2 Basic concepts of control theory	2
1.2.1 The control object	2
1.2.2 Control algorithm	3
1.2.3 Control objective	5
1.3 Modelling of control objects and their general characteristics	7
1.3.1 State equations of discrete processes	7
1.3.2 Observability and controllability	8
1.3.3 Linear proces	8
1.4 Precising the statement of the control problem	11
1.4.1 Classification of control objectives	11
1.4.2 Optimisation of control	11
1.4.3 Observations on selection of control strategies	12
<b>Chapter 2</b> Finite time period control	13
2.1 Dynamic programming	13
2.1.1 Statement of the optimization problem	13
2.1.2 Description of the Dynamic programming methods	13
2.1.3 Bellman's equation	15
2.1.4 Example: Linear-quadratic deterministic system	16
2.1.5 Generalisation of Bellman's equation for infinite time control problems	17
2.2 Stochastic control systems	18
2.2.1 Statement of the problem	18
2.2.2 Dependence of the optimal solution on the choice of the admissible control strategies	19
2.3 Stochastic dynamic programming	21
2.3.1 Description of the method	21
2.3.2 Bellman's equation for stochastic control systems	22
2.3.3 Example: Linear quadratic problem with randomly varying coefficients and observable states of the control object	23
2.3.4 Example: Linear stationary object with control delay	24
2.4 Bayesian control strategy	26
2.4.1 Bayesian approach to the optimization problem	27
2.4.2 <i>A posteriori</i> distribution and Bayesian formula	28
2.4.3 Regularity in Bayesian control strategy	30
2.4.4 Recursive formulae for computations of <i>a posteriori</i> distributions	31
2.5 Linear quadratic Gaussian Problem	33
2.5.1 Statement of the problem	33
2.5.2 Conditional Gaussism of the states and sufficient statistics	34

	2.5.3 Bayesian control strategy	35
2.A	Appendix	36
	2.A.1 General forms of probability theory	36
	2.A.2 Convergence of random variables	37
2.P	Proofs of lemmas and theorems	38
	2.P.1 Proof of the theorem 2.1.1	38
	2.P.2 Proof of the theorem 2.1.2	39
	2.P.3 Proof of the theorem 2.3.1	39
	2.P.4 Proof of the lemma 2.3.1	40
	2.P.5 Proof of the theorem 2.3.2.	40
	2.P.6 Proof of the lemma 2.4.1	40
	2.P.7 Proof of the theorem 2.4.1	41
	2.P.8 Proof of the theorem 2.4.2.	42
Chapter 3	Infinite time period control	43
3.1	Stabilization of dynamic systems using Liapunov's method	43
	3.1.1 Description of Liapunov's method	44
	3.1.2 Stabilization of linear systems with observable states	45
	3.1.3 Stabilization of linear systems with unobservable states	49
3.2	Discrete form for analytical design of regulators	51
	3.2.1 Statement of the problem	51
	3.2.2 Reduction of the optimization problem to the solvability of the matrix Riccati equation	52
	3.2.3 Lur'e equation and a few of its properties	53
	3.2.4 Analytical design of regulators in the presence of additive noise	58
3.3	Transfer function method in linear optimization problem	61
	3.3.1 Statement of the linear optimization problem	61
	3.3.2 Transfer functions of control systems and their properties	62
	3.3.3 Geometrical interpretation of the linear optimization problem	64
	3.3.4 Weiner – Kolmogorov method for conditional minimization of a quadratic functional	64
	3.3.5 Parametrization of the set of transfer functions	65
	3.3.6 Design of the optimal regulator for the object expressed in the standard form	66
	3.3.7 Correspondence between transfer function method and method of Lur'e solving equation	69
	3.3.8 Design of the optimal regulator for control object equations expressed through 'input-output' variables	69
3.4	Limiting optimal control of stochastic processes	74
	3.4.1 Sufficient conditions for optimality of admissible control strategies	75
	3.4.2 Statement of the limiting linear quadratic optimal control problem	76
	3.4.3 Solvability of the optimization problem	77
	3.4.4 Design of optimal linear regulators through transfer function method	83
	3.4.5 Formulation of the limited optimal control problems using Riccati equation	94
3.5	Minimax control	96

3.5.1	Statement of the minimax control problem	96
3.5.2	Control system transfer function and its properties	97
3.5.3	Geometrical interpretation of the minimax control problem	98
3.5.4	Properties of the sets in geometrical interpretation of the optimization problem	100
3.5.5	Statement of the basic result	103
3.5.6	The Properties of the optimal regulator	104
3.5.7	A few generalisations	105
3.A	Appendix	108
3.A.1	Frequency theorem	108
3.P	Proofs of the lemmas and theorems	109
3.P.1	Proof of the theorem 3.1.1	109
3.P.2	Proof of the theorem 3.1.2	109
3.P.3	Proof of the theorem 3.1.3	110
3.P.4	Proof of the theorem 3.1.4	110
3.P.5	Proof of the theorem 3.2.1	111
3.P.6	Proof of the theorem 3.2.2	112
3.P.7	Proof of the theorem 3.3.1	115
3.P.8	Proof of the lemma 3.3.1	118
3.P.9	Proof of the theorem 3.3.2	119
3.P.10	Proof of the lemma 3.3.2	120
3.P.11	Proof of the theorem 3.3.3	122
3.P.12	Proof of the lemma 3.3.3	122
3.P.13	Proof of the lemma 3.3.4	123
3.P.14	Proof of the theorem 3.4.1	124
3.P.15	Proof of the theorem 3.4.2	124
3.P.16	Proof of the theorem 3.4.3	125
3.P.17	Proof of the theorem 3.4.4	127
3.P.18	Proof of the theorem 3.4.5	127
3.P.19	Proof of the theorem 3.4.1	130
3.P.20	Proof of the lemma 3.4.2	130
3.P.21	Proof of the theorem 3.4.6	132
3.P.22	Proof of the theorem 3.4.7	133
3.P.23	Proof of the lemma 3.4.3	135
3.P.24	Proof of the theorem 3.5.1	136
3.P.25	Proof of the theorem 3.5.2	137
Chapter 4	Adaptive linear control systems with bounded noise	138
4.1	Fundamentals of adaptive control	140
4.1.1	Adaptive control strategy	140
4.1.2	Identification method in adaptive control	141
4.2	Existence of adaptive control strategy in a minimax control problem	143
4.2.1	Statement of the problem	144
4.2.2	Synthesis of an adaptive control strategy	146
4.2.3	Examples	148
4.3	Self-tuning systems	149

	4.3.1	Self-tuning with no disturbance	150
	4.3.2	Self-tuning in the presence of disturbance	152
	4.3.3	Adaptive control with bounded disturbance in the control object	155
	4.3.4	Method of recursive objective inequalities in an adaptive tracking problem	158
4.P		Proofs of the lemmas and theorems	162
	4.P.1	Proof of the lemma 4.2.1	162
	4.P.2	Proof of the theorem 4.2.1.	163
	4.P.3	Proof of the theorem 4.3.1	164
	4.P.4	Proof of the theorem 4.3.2	165
	4.P.5	Proof of the theorem 4.3.3	167
	4.P.6	Proof of the theorem 4.3.4	168
Chapter 5		The problem of dynamic system identification	169
5.1		Optimal recursive estimation	169
	5.1.1	Formulation of the estimation problems	170
	5.1.2	Duality of the estimation and optimal control problems	171
	5.1.3	Solution of the matrix linear quadratic cost optimization problem	172
	5.1.4	The Kalman-Bucy filter	173
	5.1.5	Optimal properties of the Kalman-Bucy filter	174
5.2		The Kalman-Bucy filter for tracking the parameter drift in dynamic systems	177
	5.2.1	Optimal tracking of the parameter drift in presence of Gaussian disturbances	178
	5.2.2	Asymptotic properties of the Kalman-Bucy filter	180
5.3		Recursive estimation	184
	5.3.1	Forecasting methods of identification	184
	5.3.2	Selection of forecasting models	185
	5.3.3	Recursive schemes for estimation	190
5.4		Identification of a linear control object in the presence of correlated noise	192
	5.4.1	Uniqueness of the minimum of the forecasting performance criterion	193
	5.4.2	Modification of the estimation algorithm	194
	5.4.3	Consistency of the estimates of the identification algorithm	195
	5.4.4	Identification of linear systems with known spectral density of noise	196
5.5		Identification of control objects using test signals	198
	5.5.1	Statement of the identification problem	198
	5.5.2	Introduction of the estimation parameter	199
	5.5.3	Estimation algorithm	199
	5.5.4	Consistency of the estimates	200
5.P		Proofs of lemmas and theorems	201
	5.P.1	Proof of the theorem 5.1.1	201
	5.P.2	Proof of the lemma 5.1.1	202
	5.P.3	Proof of the lemma 5.1.2	202
	5.P.4	Proof of the lemma 5.1.3	203
	5.P.5	Proof of the theorem 5.2.1	204
	5.P.6	Proof of the theorem 5.2.2	207
	5.P.7	Proof of the theorem 5.2.3	208

	5.P.8	Proof of the theorem 5.2.4	209
	5.P.9	Proof of the lemma 5.4.1	212
	5.P.10	Proof of the lemma 5.4.2	213
	5.P.11	Proof of the theorem 5.4.1	215
	5.P.12	Proof of the theorem 5.4.2	216
	5.P.13	Proof of the lemma 5.5.1	217
	5.P.14	Proof of the theorem 5.5.1	218
Chapter 6		Adaptive control of stochastic systems	221
6.1		Dual control	221
	6.1.1	Bayesian approach to adaptive control problems	223
	6.1.2	Adaptive version of the Gaussian linear quadratic control problems, with observable vector states	225
	6.1.3	Bayesian control strategy	231
	6.1.4	Recursive relations for <i>a posteriori</i> distributions	236
6.2		Initial synthesis of adaptive control strategy in presence of the correlated noise	243
	6.2.1	Adaptive optimal control for a performance criterion dependent on events	243
	6.2.2	Direct method of adaptive control formulation	250
	6.2.3	Adaptive optimization of the unconditional performance criterion	255
6.3		Design of the adaptive minimax control	259
	6.3.1	Statement of the problem	260
	6.3.2	Formulation of the adaptive control strategy	261
6.P		Proofs of the lemmas and the theorems	261
	6.P.1	Proof of the theorem 6.1.1	261
	6.P.2	Proof of the theorem 6.1.2	262
	6.P.3	Proof of the lemma 6.2.1	262
	6.P.4	Proof of the lemma 6.2.2	266
	6.P.5	Proof of the lemma 6.2.3	266
	6.P.6	Proof of the lemma 6.2.4	267
	6.P.7	Proof of the lemma 6.2.5	268
	6.P.8	Proof of the theorem 6.2.1	269
	6.P.9	Proof of the theorem 6.2.2	277
	6.P.10	Proof of the theorem 6.2.3	278
	6.P.11	Proof of the lemma 6.2.6	279
	6.P.12	Proof of the theorem 6.2.4	280
	6.P.13	Proof of the lemma 6.2.7	281
	6.P.14	Proof of the theorem 6.2.5	282
	6.P.15	Proof of the theorem 6.3.1	283
		COMMENTS	285
		REFERENCES	289
		OPERATORS AND NOTATIONAL CONVENTIONS	298
		SUBJECT INDEX	300