Contents

I Fourier Analysis 1

1 Fourier Series 3

1.1 Periodic Functions 3

1.2 Exponentials 5

1.3 The Bessel Inequality 7

1.4 Completeness in the L^2-Norm 8

1.5 Uniform Convergence of Fourier Series 14

1.6 Periodic Functions Revisited 16

1.7 Exercises 17

2 Hilbert Spaces 21

2.1 Pre-Hilbert and Hilbert Spaces 21

2.2 ℓ^2-Spaces 25

2.3 Orthonormal Bases and Completion 28

2.4 Fourier Series Revisited 33

2.5 Exercises 34

3 The Fourier Transform 37

3.1 Convergence Theorems 37

3.2 Convolution 39

3.3 The Transform 42

3.4 The Inversion Formula 44
3.5 Plancherel's Theorem 47
3.6 The Poisson Summation Formula 49
3.7 Theta Series 51
3.8 Exercises 52

II LCA Groups **55**

4 Finite Abelian Groups **57**
4.1 The Dual Group 57
4.2 The Fourier Transform 59
4.3 Convolution 61
4.4 Exercises 62

5 LCA Groups **65**
5.1 Metric Spaces and Topology 65
5.2 LCA Groups 72
5.3 Exercises 74

6 The Dual Group **79**
6.1 The Dual as LCA Group 79
6.2 Pontryagin Duality 85
6.3 Exercises 86

7 Plancherel Theorem **89**
7.1 Haar Integration 89
7.2 Fubini's Theorem 94
7.3 Convolution 97
7.4 Plancherel's Theorem 100

7.5 Exercises 102

III Noncommutative Groups **105**

8 Matrix Groups **107**

8.1 $GL_n(\mathbb{C})$ and $U(n)$ 107

8.2 Representations 109

8.3 The Exponential 110

8.4 Exercises 116

9 The Representations of SU(2) **119**

9.1 The Lie Algebra 120

9.2 The Representations 124

9.3 Exercises 125

10 The Peter-Weyl Theorem **127**

10.1 Decomposition of Representations 127

10.2 The Representation on $\mathrm{Hom}(\gamma, \tau)$ 128

10.3 The Peter-Weyl Theorem 129

10.4 A Reformulation 132

10.5 Exercises 133

A The Riemann Zeta Function **135**

B Haar Integration **139**

Bibliography **147**

Index **150**