

Table of Contents

Preface	VII
Introduction	XIII
Chapter 1. The Concept of a Spherical Function	1
§ 1.1. Review of Some Basic Notions of Representation Theory	1
§ 1.2. Decomposition of a Representation with Respect to a Compact Subgroup K and K -finite Representations	9
§ 1.3. Elementary Spherical Functions of Arbitrary Type	21
§ 1.4. Spherical Functions on Lie Groups	32
§ 1.5. Gelfand Pairs (G, K)	35
§ 1.6. Plancherel Formula for G/K	39
§ 1.7. Eigenfunction Expansions in G/K	49
Notes on Chapter 1	56
Chapter 2. Structure of Semisimple Lie Groups and Differential Operators on Them	58
§ 2.1. Groups of Class \mathcal{H}	58
§ 2.2. Iwasawa Decomposition. Roots. Weyl Group	61
§ 2.3. Parabolic Subalgebras and Parabolic Subgroups	65
§ 2.4. Integral Formulae	70
§ 2.5. Flag Manifolds, Bruhat Decomposition and Related Integral Formulae	76
§ 2.6. Differential Operators on G and G/K	84
Notes on Chapter 2	99
Chapter 3. The Elementary Spherical Functions	101
§ 3.1. Principal Series Representations and Integral Representations for Their Matrix Coefficients	101
§ 3.2. Determination of All Elementary Spherical Functions. The Functional Equations	104

§ 3.3. The Harish-Chandra Transform	106
§ 3.4. Finite Dimensional Representation Theory of G and Its Consequences for the H -Function and the Elementary Spherical Functions	110
§ 3.5. Convexity Properties of the H -Function	119
Notes on Chapter 3	122
 Chapter 4. The Harish-Chandra Series for φ_λ and the \mathfrak{c} -Function	 124
§ 4.1. Radial Components of Spherical Differential Operators on A^+	125
§ 4.2. The Radial Component of the Casimir Operator	132
§ 4.3. Construction of the Eigenfunctions on G^+	136
§ 4.4. The Harish-Chandra Series for φ_λ and the \mathfrak{c} -Function	146
§ 4.5. Estimates for the Harish-Chandra Series When λ Becomes Unbounded	153
§ 4.6. Estimates for the Elementary Spherical Functions. The Functions Ξ and σ	157
§ 4.7. The \mathfrak{c} -Function	169
Notes on Chapter 4	189
 Chapter 5. Asymptotic Behaviour of Elementary Spherical Functions	 192
§ 5.1. The Case When $\text{rk}(G/K) = 1$	193
§ 5.2. The Basic Differential Equations Viewed as a Perturbation of a Linear System: The Regular Case	197
§ 5.3. Radial Components on M'_{10} and M^+_{10}	200
§ 5.4. The Basic Differential Equations Viewed as a Perturbation of a Linear System: The General Case	210
§ 5.5. Spectral Theory of Representations of Polynomial Rings Associated to Finite Reflexion Groups	213
§ 5.6. The Initial Estimates	222
§ 5.7. Perturbations of Linear Systems (with A Priori Estimates)	225
§ 5.8. Asymptotics of $\Phi_0(\lambda: \cdot)$ on M^+_{10} . The Function Θ	230
§ 5.9. Asymptotics of $\varphi(\lambda: \cdot)$	240
§ 5.10. Complements. Constant Term for Tempered \mathfrak{J} -Finite Functions ...	244
Notes on Chapter 5	247
 Chapter 6. The L^2 -Theory. The Harish-Chandra Transform on the Schwartz Space of $G//K$	 249
§ 6.1. The Schwartz Spaces $\mathcal{C}(G)$ and $\mathcal{C}(G//K)$	252
§ 6.2. The Harish-Chandra Transform on $\mathcal{C}(G//K)$	262
§ 6.3. Wave Packets in $\mathcal{C}(G//K)$	268
§ 6.4. Statements of the Main Theorems	273
§ 6.5. The Method of Harish-Chandra	274
§ 6.6. The Method of Gangolli-Helgason-Rosenberg	288
Notes on Chapter 6	298

Chapter 7. L^p -Theory of Harish-Chandra Transform. Fourier Analysis on the Spaces $\mathcal{C}^p(G//K)$	300
§ 7.1. Radial Components and Their Expansions	303
§ 7.2. The Differential Equations, Initial Estimates, and the Approximating Sequence	309
§ 7.3. Expressions for $\Phi^0 - \Phi_n^0$, $\Phi_n^0 - \Phi_{n-1}^0$, and Estimates for $\Phi_0 - \exp(\Gamma_0)\Phi_n^0$	313
§ 7.4. Further Study of the Φ_n^0 . The Matrices Ω_q	315
§ 7.5. The Functions Θ_q	321
§ 7.6. Asymptotic Expansions for φ_λ	326
§ 7.7. The Tube Domains \mathcal{F}^ϵ , $^*\mathcal{F}^\epsilon$ and the Function Spaces $\mathcal{L}(\mathcal{F}^\epsilon)$, $\overline{\mathcal{L}}(\mathcal{F}^\epsilon)$	328
§ 7.8. The Spaces $\mathcal{C}^p(G//K)$	338
§ 7.9. Study of the Functions ψ_q	341
§ 7.10. Wave Packets and the Transform Theory for $\mathcal{C}^p(G//K)$	345
Notes on Chapter 7	355
Bibliography	357
Index	363