

CONTENTS

Preface	xi
1 One-Dimensional Problems—Separation of Variables	1
1.1 Introduction	1
1.2 One-Dimensional Heat Conduction	4
1.2.1 Boundary and Initial Conditions	6
1.3 Steady State Solutions	6
1.4 Time Dependent Heat Flow—Separation of Variables	10
1.4.1 Elementary Solutions	11
1.4.2 Synthesis of Elementary Solutions—Fourier Series	13
1.4.3 Changing the Boundary Conditions—Insulated Ends	17
1.5 Steady State Heating by a Localized Source—Delta Function	21
1.6 Inhomogeneous Boundary Conditions	28
1.7 Inhomogeneous Heat Equation—Source Terms	33
1.8 Wave Equation—Vibrating String	38
1.8.1 General Wave Equation	39
1.8.2 Solution by Separation of Variables	41
1.8.3 Energy Flow on the String	45
1.8.4 Source Terms—Inhomogeneous Wave Equation	46
1.9 d’Alembert’s Solution of the Wave Equation	49
2 Laplace Transform Method	60
2.1 Vibrating String	60
2.2 Diffusion Equation	65
2.3 Miscellaneous Examples	74
2.3.1 Impulse Acting on a String	74
2.3.2 Long-Time Behavior	75
2.3.3 Total Heat Flow through an End	77
2.4 Point Source Problem—Preview of Green’s Function	79
3 Two and Three Dimensions	85
3.1 Introduction	85
3.2 Steady State Temperature Distribution in Rectangular Coordinates—Laplace’s Equation	86

3.3	Time Dependent Diffusion in Rectangular Coordinates	93
3.4	Waves on a Membrane—Rectangular Coordinates	98
3.4.1	Normal Modes	99
3.4.2	Guided Waves	101
3.5	Orthogonal Curvilinear Coordinates	105
3.5.1	Cylindrical Coordinates	108
3.5.2	Spherical Coordinates	108
3.5.3	Oblate Spheroidal Coordinates	111
3.6	Spherical Symmetry	114
3.6.1	Spherical Standing Waves	115
3.6.2	Spherical Traveling Waves	116
3.7	Circular and Cylindrical Symmetry	118
3.7.1	Steady State Temperature in a Pie-Shaped Region	118
3.7.2	Laplace's Equation in an Annular Circle	121
3.7.3	Vibrating Membrane	126
3.7.4	Steady State Temperature in a Cylinder	131
4	Green's Functions	142
4.1	Introductory Example	142
4.1.1	Reciprocity	145
4.2	General Procedure for Constructing Green's Functions in One Dimension	155
4.3	One-Dimensional Steady Waves	158
4.3.1	Scattering of Waves on a String	160
4.3.2	Significance of Boundary Terms	162
4.4	Method of Images	165
4.5	A Non-Self-adjoint Green's Function	167
4.6	Green's Function for a Damped Oscillator	174
4.6.1	A Generalization	177
4.7	One-Dimensional Diffusion and Wave Motion	179
4.7.1	Diffusion Equation	179
4.7.2	Wave Equation	183
4.8	Two and Three Space Dimensions—Green's Theorem	188
4.9	Green's Function in Free Space	191
4.9.1	Boundary Condition at Infinity	192
4.10	Two-Dimensional Problems	199
4.11	Inversion and the Method of Images for a Circle	205
4.12	Eigenfunction Expansion Methods	210
4.13	Modified Green's Functions—One Dimension	221

5	Spherical Geometry	226
5.1	Solution of Laplace's Equation	226
5.2	Source Terms and the Multiple Expansion	235
5.2.1	Axial Multipoles	236
5.3	Inversion—Green's Function for a Sphere	240
5.4	Spherical Waves	245
5.4.1	Spherical Bessel Functions	247
5.4.2	Radiation from a Point Source	248
5.4.3	Reduction to a Plane Wave	250
5.4.4	Scattering of a Plane Wave by a Sphere	251
6	Fourier Transform Methods	255
6.1	Fourier Sine and Cosine Transforms	255
6.2	Examples	258
6.2.1	Green's Function for Steady Waves on a Semi-infinite String	258
6.2.2	Temperature Distribution in a Quarter Plane	260
6.2.3	d'Alembert Solution for a Semi-infinite String	262
6.3	Convolution Theorems	266
6.4	Complex Fourier Transforms	272
6.4.1	Approach to the Boundary	276
6.5	Fourier Transforms in Two and Three Dimensions	281
6.6	Circular Symmetry, Fourier-Bessel Transform	286
6.7	Green's Functions for Time Dependent Wave Equation in One, Two, and Three Dimensions	291
6.7.1	One Dimension	291
6.7.2	Two Dimensions	293
6.7.3	Three Dimensions	294
7	Perturbation Methods	297
7.1	First Order Corrections	297
7.2	Equal Frequencies (Degeneracy)	303
7.3	Variational Methods	307
7.3.1	Differential Equation Approach (Rayleigh's Method)	307
7.3.2	Integral Equation Approach	312
8	Generalizations and First Order Equations	317
8.1	Classification of Second Order Equations	317
8.2	Uniqueness and General Properties of Solutions	325
8.2.1	Laplace's Equations	326

8.2.2	Uniqueness	327
8.2.3	Diffusion Equation	327
8.2.4	Wave Equation	328
8.3	First Order Equations	331
8.4	Burger's Equation	343
9	Selected Topics	352
9.1	Oscillating Heat Source on a Beam	352
9.2	Temperature Distribution in a Pie-Shaped Region	356
9.3	Babinet's Principle	361
9.4	Comparison of Wave Motion in One, Two, and Three Dimensions—Fractional Derivatives	364
9.5	Modified Green's Function for a Sphere	370
9.6	Oscillation of an Inhomogeneous Chain	373
9.7	Point Source Near the Interface between Two Half Spaces	376
9.8	Waves in an Inhomogeneous Medium	379
9.9	A Hybrid Fourier Transform	382
9.10	Invariants of the Linear Parabolic Equation	387
Appendix A	Fourier Series	391
Appendix B	Laplace Transform	410
Appendix C	Sturm-Liouville Equations	426
Appendix D	Bessel Functions	436
Appendix E	Legendre Polynomials	450
Appendix F	Tables of Sums and Integral Transforms	462
References		466
Index		467