

Table of Contents

1	Overview and Basic Concepts	1
1.1	Introduction	1
1.2	The Programs	5
1.3	Literature on Chaotic Dynamics	8
2	Nonlinear Dynamics and Deterministic Chaos	11
2.1	Deterministic Chaos	11
2.2	Hamiltonian Systems	12
2.2.1	Integrable and Ergodic Systems	13
2.2.2	Poincaré Sections	16
2.2.3	The KAM Theorem	18
2.2.4	Homoclinic Points	20
2.3	Dissipative Dynamical Systems	21
2.3.1	Attractors	23
2.3.2	Routes to Chaos	25
2.4	Special Topics	27
2.4.1	The Poincaré-Birkhoff Theorem	27
2.4.2	Continued Fractions	29
2.4.3	The Lyapunov Exponent	31
2.4.4	Fixed Points of One-Dimensional Maps	34
2.4.5	Fixed Points of Two-Dimensional Maps	37
2.4.6	Bifurcations	42
	References	43
3	Billiard Systems	45
3.1	Deformations of a Circle Billiard	47
3.2	Numerical Techniques	51
3.3	Interacting with the Program	52
3.4	Computer Experiments	56
3.4.1	From Regularity to Chaos	56
3.4.2	Zooming In	58
3.4.3	Sensitivity and Determinism	60
3.4.4	Suggestions for Additional Experiments	61
	Stability of Two-Bounce Orbits	61

	Bifurcations of Periodic Orbits	62
	A New Integrable Billiard?	64
	Non-Convex Billiards	64
3.5	Suggestions for Further Studies	64
3.6	Real Experiments and Empirical Evidence	65
	References	65
4	Gravitational Billiards: The Wedge	67
4.1	The Poincaré Mapping	68
4.2	Interacting with the Program	73
4.3	Computer Experiments	75
4.3.1	Periodic Motion and Phase Space Organization	75
4.3.2	Bifurcation Phenomena	80
4.3.3	'Plane Filling' Wedge Billiards	84
4.3.4	Suggestions for Additional Experiments	86
	Mixed $A-B$ Orbits	86
	Pure B Dynamics	87
	The Stochastic Region	87
	Breathing Chaos	87
4.4	Suggestions for Further Studies	87
4.5	Real Experiments and Empirical Evidence	88
	References	88
5	The Double Pendulum	89
5.1	Equations of Motion	89
5.2	Numerical Algorithms	91
5.3	Interacting with the Program	93
5.4	Computer Experiments	98
5.4.1	Different Types of Motion	98
5.4.2	Dynamics of the Double Pendulum	103
5.4.3	Destruction of Invariant Curves	107
5.4.4	Suggestions for Additional Experiments	110
	Testing the Numerical Integration	110
	Zooming In	110
	Different Pendulum Parameters	111
5.5	Real Experiments and Empirical Evidence	112
	References	114
6	Chaotic Scattering	115
6.1	Scattering off Three Disks	117
6.2	Numerical Techniques	121
6.3	Interacting with the Program	122
6.4	Computer Experiments	124
6.4.1	Scattering Functions and Two-Disk Collisions	124
6.4.2	Tree Organization of Three-Disk Collisions	127
6.4.3	Unstable Periodic Orbits	129

6.4.4	Fractal Singularity Structure	131
6.4.5	Suggestions for Additional Experiments	133
	Long-Lived Trajectories	133
	Incomplete Symbolic Dynamics	135
	Multiscale Fractals	135
6.5	Suggestions for Further Studies	135
6.6	Real Experiments and Empirical Evidence	135
	References	136
7	Fermi Acceleration	137
7.1	Fermi Mapping	138
7.2	Interacting with the Program	139
7.3	Computer Experiments	141
	7.3.1 Exploring Phase Space for Different Wall Oscillations . .	141
	7.3.2 KAM Curves and Stochastic Acceleration	144
	7.3.3 Fixed Points and Linear Stability	147
	7.3.4 Absolute Barriers	148
	7.3.5 Suggestions for Additional Experiments	150
	Higher Order Fixed Points	150
	Standard Mapping	151
	Bifurcation Phenomena	152
	Influence of Different Wall Velocities	153
7.4	Suggestions for Further Studies	154
7.5	Real Experiments and Empirical Evidence	155
	References	155
8	The Duffing Oscillator	157
8.1	The Duffing Equation	157
8.2	Numerical Techniques	161
8.3	Interacting with the Program	161
8.4	Computer Experiments	165
	8.4.1 Chaotic and Regular Oscillations	165
	8.4.2 The Free Duffing Oscillator	166
	8.4.3 Anharmonic Vibrations: Resonances and Bistability . . .	166
	8.4.4 Coexisting Limit Cycles and Strange Attractors	170
	8.4.5 Suggestions for Additional Experiments	173
	Harmonic Oscillator	173
	Gravitational Pendulum	174
	Exact Harmonic Response	174
	Period-Doubling Bifurcations	175
	Strange Attractors	177
8.5	Suggestions for Further Studies	177
8.6	Real Experiments and Empirical Evidence	178
	References	179