

# Contents

<b>Preface</b>	<b>vii</b>
<b>Acknowledgments</b>	<b>xi</b>
<b>Standard Notation</b>	<b>xxiii</b>
<b>1 Linear vs. Nonlinear</b>	<b>1</b>
1.1 Nonlinear Models . . . . .	1
1.2 Complexity in Nonlinear Dynamics . . . . .	4
1.2.1 Subtleties of Nonlinear Systems Analysis . . . . .	10
1.2.2 Autonomous Systems and Equilibrium Points . . . . .	12
1.3 Some Classical Examples . . . . .	14
1.3.1 The Tunnel Diode Circuit . . . . .	14
1.3.2 An Oscillating Circuit: Due to van der Pol . . . . .	16
1.3.3 The Pendulum: Due to Newton . . . . .	17
1.3.4 The Buckling Beam: Due to Euler . . . . .	19
1.3.5 The Volterra–Lotka Predator–Prey Equations . . . . .	22
1.4 Other Classics: Musical Instruments . . . . .	23
1.4.1 Blowing of a Clarinet Reed: Due to Rayleigh . . . . .	23
1.4.2 Bowing of a Violin String: Due to Rayleigh . . . . .	23
1.5 Summary . . . . .	25
1.6 Exercises . . . . .	26
<b>2 Planar Dynamical Systems</b>	<b>31</b>
2.1 Introduction . . . . .	31
2.2 Linearization About Equilibria of Second-Order Nonlinear Systems . . . . .	31

2.2.1	Linear Systems in the Plane . . . . .	32
2.2.2	Phase Portraits near Hyperbolic Equilibria . . . . .	36
2.3	Closed Orbits of Planar Dynamical Systems . . . . .	41
2.4	Counting Equilibria: Index Theory . . . . .	49
2.5	Bifurcations . . . . .	51
2.6	Bifurcation Study of Josephson Junction Equations . . . . .	55
2.7	The Degenerate van der Pol Equation . . . . .	60
2.8	Planar Discrete-Time Systems . . . . .	63
2.8.1	Fixed Points and the Hartman–Grobman Theorem . . . . .	63
2.8.2	Period $N$ Points of Maps . . . . .	65
2.8.3	Bifurcations of Maps . . . . .	66
2.9	Summary . . . . .	69
2.10	Exercises . . . . .	69
<b>3</b>	<b>Mathematical Background</b>	<b>76</b>
3.1	Groups and Fields . . . . .	76
3.2	Vector Spaces, Algebras, Norms, and Induced Norms . . . . .	77
3.3	Contraction Mapping Theorems . . . . .	82
3.3.1	Incremental Small Gain Theorem . . . . .	84
3.4	Existence and Uniqueness Theorems for Ordinary Differential Equations . . . . .	86
3.4.1	Dependence on Initial Conditions on Infinite Time Intervals . . . . .	90
3.4.2	Circuit Simulation by Waveform Relaxation . . . . .	90
3.5	Differential Equations with Discontinuities . . . . .	93
3.6	Carleman Linearization . . . . .	99
3.7	Degree Theory . . . . .	101
3.8	Degree Theory and Solutions of Resistive Networks . . . . .	105
3.9	Basics of Differential Topology . . . . .	107
3.9.1	Smooth Manifolds and Smooth Maps . . . . .	107
3.9.2	Tangent Spaces and Derivatives . . . . .	110
3.9.3	Regular Values . . . . .	113
3.9.4	Manifolds with Boundary . . . . .	115
3.10	Summary . . . . .	118
3.11	Exercises . . . . .	119
<b>4</b>	<b>Input–Output Analysis</b>	<b>127</b>
4.1	Optimal Linear Approximants to Nonlinear Systems . . . . .	128
4.1.1	Optimal Linear Approximations for Memoryless, Time-Invariant Nonlinearities . . . . .	132
4.1.2	Optimal Linear Approximations for Dynamic Nonlinearities: Oscillations in Feedback Loops . . . . .	135
4.1.3	Justification of the Describing Function . . . . .	138
4.2	Input–Output Stability . . . . .	143
4.3	Applications of the Small Gain Theorems . . . . .	150

4.3.1	Robustness of Feedback Stability . . . . .	150
4.3.2	Loop Transformation Theorem . . . . .	152
4.4	Passive Nonlinear Systems . . . . .	153
4.5	Input–Output Stability of Linear Systems . . . . .	156
4.6	Input–Output Stability Analysis of Feedback Systems . . . . .	160
4.6.1	The Lur’e Problem . . . . .	162
4.7	Volterra Input–Output Representations . . . . .	167
4.7.1	Homogeneous, Polynomial and Volterra Systems in the Time Domain . . . . .	168
4.7.2	Volterra Representations from Differential Equations . . . . .	170
4.7.3	Frequency Domain Representation of Volterra Input Output Expansions . . . . .	173
4.8	Summary . . . . .	174
4.9	Exercises . . . . .	175
<b>5</b>	<b>Lyapunov Stability Theory</b>	<b>182</b>
5.1	Introduction . . . . .	182
5.2	Definitions . . . . .	183
5.2.1	The Lipschitz Condition and Consequences . . . . .	183
5.3	Basic Stability Theorems of Lyapunov . . . . .	188
5.3.1	Energy-Like Functions . . . . .	188
5.3.2	Basic Theorems . . . . .	189
5.3.3	Examples of the Application of Lyapunov’s Theorem . . . . .	192
5.3.4	Exponential Stability Theorems . . . . .	195
5.4	LaSalle’s Invariance Principle . . . . .	198
5.5	Generalizations of LaSalle’s Principle . . . . .	204
5.6	Instability Theorems . . . . .	206
5.7	Stability of Linear Time-Varying Systems . . . . .	207
5.7.1	Autonomous Linear Systems . . . . .	209
5.7.2	Quadratic Lyapunov Functions for Linear Time Varying Systems . . . . .	212
5.8	The Indirect Method of Lyapunov . . . . .	214
5.9	Domains of Attraction . . . . .	217
5.10	Summary . . . . .	223
5.11	Exercises . . . . .	223
<b>6</b>	<b>Applications of Lyapunov Theory</b>	<b>235</b>
6.1	Feedback Stabilization . . . . .	235
6.2	The Lur’e Problem, Circle and Popov Criteria . . . . .	237
6.2.1	The Circle Criterion . . . . .	240
6.2.2	The Popov Criterion . . . . .	245
6.3	Singular Perturbation . . . . .	247
6.3.1	Nonsingular Points, Solution Concepts, and Jump Behavior . . . . .	250
6.4	Dynamics of Nonlinear Systems and Parasitics . . . . .	252

6.4.1	Dynamics of Nonlinear Circuits . . . . .	252
6.4.2	Dynamics of Power Systems . . . . .	255
6.5	Adaptive Identification of Single-Input Single-Output Linear Time-Invariant Systems . . . . .	256
6.5.1	Linear Identifier Stability . . . . .	262
6.5.2	Parameter Error Convergence . . . . .	265
6.6	Averaging . . . . .	266
6.7	Adaptive Control . . . . .	273
6.8	Back-stepping Approach to Stabilization . . . . .	275
6.9	Summary . . . . .	277
6.10	Exercises . . . . .	278
<b>7</b>	<b>Dynamical Systems and Bifurcations</b>	<b>287</b>
7.1	Qualitative Theory . . . . .	287
7.2	Nonlinear Maps . . . . .	291
7.3	Closed Orbits, Poincaré Maps, and Forced Oscillations . . . . .	294
7.3.1	The Poincaré Map and Closed Orbits . . . . .	294
7.3.2	The Poincaré Map and Forced Oscillations . . . . .	297
7.4	Structural Stability . . . . .	302
7.5	Structurally Stable Two Dimensional Flows . . . . .	305
7.6	Center Manifold Theorems . . . . .	309
7.6.1	Center Manifolds for Flows . . . . .	309
7.6.2	Center Manifolds for Flows Depending on Parameters . . . . .	313
7.6.3	Center Manifolds for Maps . . . . .	314
7.7	Bifurcation of Vector Fields: An Introduction . . . . .	315
7.8	Bifurcations of Equilibria of Vector Fields . . . . .	317
7.8.1	Single, Simple Zero Eigenvalue . . . . .	317
7.8.2	Pure Imaginary Pair of Eigenvalues: Poincaré–Andronov–Hopf Bifurcation . . . . .	322
7.9	Bifurcations of Maps . . . . .	324
7.9.1	Single Eigenvalue 1: Saddle Node, Transcritical and Pitchfork . . . . .	325
7.9.2	Single Eigenvalue $-1$ : Period Doubling . . . . .	327
7.9.3	Pair of Complex Eigenvalues of Modulus 1: Naimark–Sacker Bifurcation . . . . .	328
7.10	More Complex Bifurcations of Vector Fields and Maps . . . . .	329
7.10.1	Bifurcations of Equilibria and Fixed Points: Catastrophe Theory . . . . .	329
7.10.2	Singular Perturbations and Jump Behavior of Systems . . . . .	335
7.10.3	Dynamic Bifurcations: A Zoo . . . . .	337
7.11	Routes to Chaos and Complex Dynamics . . . . .	340
7.12	Exercises . . . . .	341
<b>8</b>	<b>Basics of Differential Geometry</b>	<b>349</b>
8.1	Tangent Spaces . . . . .	349

	8.1.1 Vector Fields, Lie Brackets, and Lie Algebras . . . . .	352
8.2	Distributions and Codistributions . . . . .	356
8.3	Frobenius Theorem . . . . .	359
8.4	Matrix Groups . . . . .	362
	8.4.1 Matrix Lie Groups and Their Lie Algebras . . . . .	367
	8.4.2 The Exponential Map . . . . .	369
	8.4.3 Canonical Coordinates on Matrix Lie Groups . . . . .	370
	8.4.4 The Campbell–Baker–Hausdorff Formula . . . . .	371
8.5	Left-Invariant Control Systems on Matrix Lie Groups . . . . .	375
	8.5.1 Frenet–Serret Equations: A Control System on $SE(3)$ . . . . .	375
	8.5.2 The Wei–Norman Formula . . . . .	378
8.6	Summary . . . . .	379
8.7	Exercises . . . . .	379
<b>9</b>	<b>Linearization by State Feedback</b>	<b>384</b>
9.1	Introduction . . . . .	384
9.2	SISO Systems . . . . .	385
	9.2.1 Input–Output Linearization . . . . .	385
	9.2.2 Zero Dynamics for SISO Systems . . . . .	398
	9.2.3 Inversion and Exact Tracking . . . . .	402
	9.2.4 Asymptotic Stabilization and Tracking for SISO Systems . . . . .	404
9.3	MIMO Systems . . . . .	407
	9.3.1 MIMO Systems Linearizable by Static State Feedback . . . . .	407
	9.3.2 Full State Linearization of MIMO Systems . . . . .	411
	9.3.3 Dynamic Extension for MIMO Systems . . . . .	414
9.4	Robust Linearization . . . . .	417
9.5	Sliding Mode Control . . . . .	423
	9.5.1 SISO Sliding Mode Control . . . . .	423
	9.5.2 MIMO Sliding Mode Control . . . . .	425
9.6	Tracking for Nonminimum Phase Systems . . . . .	425
	9.6.1 The Method of Devasia, Chen, and Paden . . . . .	426
	9.6.2 The Byrnes–Isidori Regulator . . . . .	429
9.7	Observers with Linear Error Dynamics . . . . .	433
9.8	Summary . . . . .	439
9.9	Exercises . . . . .	440
<b>10</b>	<b>Design Examples Using Linearization</b>	<b>449</b>
10.1	Introduction . . . . .	449
10.2	The Ball and Beam Example . . . . .	450
	10.2.1 Dynamics . . . . .	451
	10.2.2 Exact Input–Output Linearization . . . . .	451
	10.2.3 Full State Linearization . . . . .	453
	10.2.4 Approximate Input–Output Linearization . . . . .	454
	10.2.5 Switching Control of the Ball and Beam System . . . . .	458

10.3	Approximate Linearization for Nonregular SISO Systems . . .	460
10.3.1	Tracking for Nonregular Systems . . . . .	468
10.4	Nonlinear Flight Control . . . . .	468
10.4.1	Force and Moment Generation . . . . .	469
10.4.2	Simplification to a Planar Aircraft . . . . .	472
10.4.3	Exact Input–Output Linearization of the PVTOL Aircraft System . . . . .	473
10.4.4	Approximate Linearization of the PVTOL Aircraft . .	476
10.5	Control of Slightly Nonminimum Phase Systems . . . . .	481
10.5.1	Single Input Single Output (SISO) Case . . . . .	481
10.5.2	Generalization to MIMO Systems . . . . .	484
10.6	Singularly Perturbed Zero Dynamics for Regularly Perturbed Nonlinear Systems . . . . .	490
10.6.1	SISO Singularly Perturbed Zero and Driven Dynamics .	490
10.6.2	MIMO Singularly Perturbed Zero and Driven Dynamics . . . . .	493
10.7	Summary . . . . .	498
10.8	Exercises . . . . .	499
<b>11</b>	<b>Geometric Nonlinear Control</b>	<b>510</b>
11.1	Controllability Concepts . . . . .	510
11.2	Drift–Free Control Systems . . . . .	513
11.3	Steering of Drift–Free Nonholonomic Systems . . . . .	518
11.4	Steering Model Control Systems Using Sinusoids . . . . .	520
11.5	General Methods for Steering . . . . .	529
11.5.1	Fourier Techniques . . . . .	529
11.5.2	Optimal Steering of Nonholonomic Systems . . . . .	535
11.5.3	Steering with Piecewise Constant Inputs . . . . .	539
11.5.4	Control Systems with Drift . . . . .	545
11.6	Observability Concepts . . . . .	549
11.7	Zero Dynamics Algorithm and Generalized Normal Forms . .	551
11.8	Input–Output Expansions for Nonlinear Systems . . . . .	560
11.9	Controlled Invariant Distributions and Disturbance Decoupling .	564
11.10	Summary . . . . .	566
11.11	Exercises . . . . .	567
<b>12</b>	<b>Exterior Differential Systems in Control</b>	<b>574</b>
12.1	Introduction . . . . .	574
12.2	Introduction to Exterior Differential Systems . . . . .	575
12.2.1	Multilinear Algebra . . . . .	576
12.2.2	Forms . . . . .	592
12.2.3	Exterior Differential Systems . . . . .	600
12.3	Normal Forms . . . . .	606
12.3.1	The Goursat Normal Form . . . . .	606
12.3.2	The $n$ -trailer Pfaffian System . . . . .	614

12.3.3	The Extended Goursat Normal Form . . . . .	623
12.4	Control Systems . . . . .	629
12.5	Summary . . . . .	635
12.6	Exercises . . . . .	636
<b>13</b>	<b>New Vistas: Multi-Agent Hybrid Systems</b>	<b>641</b>
13.1	Embedded Control and Hybrid Systems . . . . .	641
13.2	Multi-Agent Systems and Hybrid Systems . . . . .	642
	<b>References</b>	<b>645</b>
	<b>Index</b>	<b>661</b>