

CONTENTS

	PAGE
PREFACE	v

CHAPTER I

THE ATTRACTION OF FINITE BODIES

SECTION		PAGE
1.	The Law of Gravitation	1
2.	The Attraction of Systems of Particles	1
3.	The Components of Attraction.	1
4.	The Elements of Mass	2
5.	Attraction on a Point.	3
6.	The Attraction of a Circular Arc on Its Center.	3
7.	The Attraction of a Straight Line on a Point.	4
8.	The Attraction of a Thin Sheet on Its Axis of Symmetry.	5
9.	The Attraction of the Frustum of a Cone on Its Apex.	7
10.	Perspectivity	8
11.	The Attraction of an Ellipsoidal Homoeoid upon an Interior Point	10
12.	The Attraction of a Spherical Shell upon an Exterior Particle	11
13.	The Attraction of a Solid Sphere upon an Exterior Point	14
14.	The Mutual Attraction of Two Straight Collinear Rods.	14
15.	The Attraction of a Circular Disk on Its Axis	15
16.	The Attraction of a Body of Revolution on a Point in Its Axis	16
17.	Example—The Oblate Spheroid	17
18.	The Attraction of a Uniform Rectangular Plate on a Point in Its Own Plane	19
19.	The Attraction between Two Rigid Bodies	21
	Problems	21

CHAPTER II

THE NEWTONIAN POTENTIAL FUNCTION

20.	The Potential Function Defined	24
21.	The Significance of the Potential Function.	25
22.	The Potential Function Exists.	26
23.	The Existence of Derivatives of the Potential	27
24.	Existence of Derivatives at Exterior Points	27
25.	Existence of Derivatives at Interior Points	29
26.	The Equation of Laplace	32
27.	Equipotential Surfaces, or Level Surfaces	34
28.	The Logarithmic Potential.	35
29.	The Potential of a Spherical Shell	36

SECTION		PAGE
30. Potential of a Uniform Circular Disk along Its Axis		41
31. Potential of a Homogeneous Straight Rod.		42
32. The Potential of a Homogeneous Solid Ellipsoid for Interior Points		45
33. The Equipotential Surfaces		50
34. The Components of Attraction at an Interior Point.		51
35. The Attraction of a Solid Homogeneous Ellipsoid upon an Exterior Point—Ivory's Method.		52
36. The Potential of a Homogeneous Solid Ellipsoid at Exterior Points		56
37. Evaluation of the Elliptic Integrals.		58
38. MacLaurin's Theorem		60
39. The Potential of Spheroids at Exterior Points		62
40. The Attraction of a Spheroid at the Surface.		63
41. The Attraction Is a Maximum.		66
42. The Potential of Homogeneous Elliptic Cylinders.		69
43. The Potential of a Homogeneous, Rectangular Parallelopiped		72
44. The Components of Force for the Right Parallelopiped		79
45. A Generalization Regarding Derivatives of a Potential		80
46. The Potential of a Body at a Distant Point		81
47. The Terms of Higher Degrees		85
48. The Expansion for the Homogeneous Ellipsoid.		87
49. The Right Parallelopiped		88
50. The Inertial Integrals.		89
51. The Inertial Integrals Cannot All Vanish		91
52. A Body Is Uniquely Defined by Its Inertial Integrals.		94
Problems		94

CHAPTER III

VECTOR FIELDS. THEOREMS OF GREEN AND GAUSS

53. Definitions		96
54. The Normal Derivative		97
55. Relations between Certain Volume- and Surface-Integrals.		99
56. A Vector Interpretation.		101
57. Generalized Orthogonal Coordinates		102
58. Green's Theorem.		104
59. The Potential of Homogeneous Bodies		106
60. Example—A Non-Homogeneous Spherical Shell		107
61. Existence of Higher Derivatives of Potential Functions		109
62. Harmonic Functions		111
63. An Extension of Green's Theorem for Harmonic Functions		111
64. Reduction to Two Dimensions.		115
65. Analogy with Cauchy's Theory of Residues		117
66. The Surface Integral of the Normal Derivatives of $1/\rho$		120
67. The Contour Integral of the Normal Derivative of $\log \rho$		122
68. A Theorem of Gauss		123
69. Poisson's Equation.		124
70. Poisson's Equation in Two Dimensions		126

SECTION	PAGE
71. An Extension of Gauss' Theorem	126
72. Green's Theorem Applied to Two Potential Functions	128
73. Characteristic Properties of a Potential Function	130
74. The Average Value of a Potential Function over a Sphere	132
75. Maxima and Minima of Harmonic Functions	133
76. The Potential Energy of a Finite Mass	136
77. The Potential Energy of a Homogeneous Sphere	138
78. The Heat of the Sun	139
79. Relation between Certain Surface and Line Integrals	140
80. Stokes' Theorem.	143
81. Examples of Vector Curls.	144
82. The Vector and Its Curl Are Orthogonal	145
83. Condition That a Line Integral Shall Be Independent of the Path of Integration	148
84. Condition That a Surface Integral Shall Depend upon the Contour Only	149
Problems	152

CHAPTER IV

THE ATTRACTIONS OF SURFACES AND LINES

85. The Occasion for Their Study	153
Attractions of Surfaces	
86. A Uniform Disk	153
87. An Infinite Homogeneous Universe.	155
88. Proper and Improper Integrals.	157
89. Semi-convergent Integrals.	161
90. The Potential at a Point of the Surface.	165
91. The Potential Is Continuous across the Surface	166
92. The Normal Component of the Attraction Is Discontinuous across the Surface	168
93. The Tangential Components of the Attraction Are Continuous	171
94. Discontinuities in the Derivatives of Surface Potentials.	174
95. Example—A Non-homogeneous Disk.	176
96. Discontinuities in the Second Derivatives of Surface Potentials .	178
97. Singular Points of the Surface	189
Attractions of Lines	
98. A Straight Rod	190
99. The Components of Attraction.	191
100. Attraction in the Line Is Not Well Defined	193
101. Asymptotic Expression for the Potential	194
102. The Potential of a Uniform Hoop	195
103. Evaluation of the Potential According to Gauss	197
104. Asymptotic Expression for the Potential	200

CHAPTER V

SURFACE DISTRIBUTIONS OF MATTER

105. Transformation by Reciprocal Radii	204
106. Application of the Transformation to Potentials	207

SECTION		PAGE
107.	The Potential of a Uniform Distribution of Matter on a Sphere	209
108.	A Non-uniform Spherical Distribution	209
109.	Inversion of a Homogeneous Ellipsoidal Shell	211
110.	Centrobaric Bodies.	212
111.	The Center of Gravity of Centrobaric Bodies	213
112.	The Central Ellipsoid of Inertia	214
113.	A System of Detached Masses Cannot Be Centrobaric	215
114.	Theorems Relating to Electric Images	216
115.	Level Layers.	219
116.	Families of Level Layers	221
117.	Level Layer on an Arbitrarily Given Surface.	222
118.	Robin's Integral Equation.	227
119.	Picard's Solution of Robin's Equation.	228
120.	Example of a Level Layer.	231
121.	Level Layers on Prolate Spheroids	233
122.	Level Layers on Ellipsoids.	235
123.	The Potential of Ellipsoidal Level Layers	239
124.	Layers of Finite Thickness.	240
125.	A Finite Shell Bounded by Confocal Spheroids.	241
126.	The Surface Density Necessary to Produce Given Potentials	244
127.	Green's Problem.	246
128.	Certain Physical Considerations	247
129.	The Existence of Green's Function.	248
130.	Miscellaneous Properties of Green's Function	249
131.	The Green Function Is Symmetric.	252
132.	The Normal Derivative of Green's Function Is Harmonic.	253
133.	The Green Function for the Sphere.	254
134.	The Normal Derivative on the Sphere	257
135.	Green's Equation for the Sphere.	260
136.	A Generalization for the Sphere	261
137.	Green's Equation for any Closed Surface	265
138.	A General Theorem of Green's.	269
	Green's Problem for the Logarithmic Potential	
139.	Statement of the Problem.	269
140.	Electric Images for the Logarithmic Potential	270
141.	Electric Images of Centrobaric Bodies	271
142.	The Existence of Green's Function.	274
143.	Green's Function for the Circle	276
144.	The Principle of Dirichlet and Lord Kelvin	277
145.	The Equivalent Problem of Poincaré	280
	Problems	282

CHAPTER VI

TWO-LAYER SURFACES

The Methods of Neumann and Poincaré

146.	Various Types of Mass	283
147.	The Magnetic Doublet	283

SECTION	PAGE
148. The Bar Magnet	285
149. Magnetic Sheets—Two-layer Surfaces	285
150. Closed, Uniform, Two-layer Surfaces	286
151. Uniform Surfaces Not Closed	288
152. Surfaces with Variable Moments	290
153. Discontinuities in the First Derivatives	292
154. The Configuration Constant of a Closed Surface	293
155. The Spread of Values of the Potential on a Closed, Convex Surface	295
156. Neumann's Proof of Dirichlet's Principle	296
157. The Limiting Values of the Potentials W_n on S	299
158. Harnack's Theorem for Harmonic Functions	300
159. Case I—The Constant C Is Zero	305
160. The Interior and Exterior Functions as Potentials of the Same Simple Layer	307
161. The Constant C Is Not Zero	309
162. The Construction of a Simple, Level Layer on S Poincaré's Méthode du Balayage	311
163. The Balayage of a Sphere	313
164. Existence of a Level Layer on a Given Surface	314
165. Application of Harnack's Theorem	318
166. Construction of an Infinite System of Spheres within S	319
167. The Existence of the Required Harmonic Function	320
Problems	323

CHAPTER VII

SPHERICAL HARMONICS

168. Definitions	325
169. Examples of Spherical Harmonics	326
170. Homogeneous, Harmonic Polynomials	326
171. Relation between Certain Harmonics	328
172. The Expansion of a Potential	328
173. Rotation about an Imaginary Axis	330
174. Harmonics Which Depend upon r and z Alone	331
175. The Equation of Laplace for Surface Harmonics	332
176. Zonal Harmonics	333
177. The Polynomials of Legendre	335
178. The Expansion in Taylor's Series	337
179. The Expansion in Lagrange's Series	338
180. Zonal Harmonics Given Explicitly	339
181. The Zeros of the Zonal Harmonics Are All Real	340
182. Certain Useful Relations	342
183. The Zonal Harmonics Are Orthogonal Functions	343
184. A Generalization of the Preceding Formulas	345
185. A Recursion Formula for Zonal Harmonics	346
186. The General Formula for Zonal Harmonics	347
187. The General Expression for $H_n^{(1)}$	349
188. Zonal Harmonics Expressed by Cosines of Multiples of the Argument	350

SECTION		PAGE
189.	Powers of μ Expressed in Terms of Zonal Harmonics	352
190.	A Definite Integral Representation of Zonal Harmonics.	355
191.	An Important Property of Zonal Harmonics.	356
192.	Expansion of $\sin m\varphi$ in a Series of Zonal Harmonics	357
193.	The Potential of a Solid of Revolution	360
194.	The Oblate Spheroid	363
195.	The Apparent Size of a Plane Circular Disk	365
196.	The Potential of a Zonal Distribution of Matter on a Spherical Surface.	366
197.	Tesseral Harmonics.	368
198.	Examples of Solid Tesseral Harmonics	371
199.	The Zeros of the Tesseral Harmonics.	372
200.	The Surface Integral of the Product of Two Spherical Harmonics of Different Degrees	373
201.	The Surface Integral of the Product of Two Spherical Harmonics of the Same Degree.	374
202.	The Expansion of $1/\rho$ in a Series of Tesseral Harmonics.	376
203.	The Expansion of the Potential of a Finite Body in a Series of Tesseral Harmonics.	380
204.	The Expansion of the Potential of a Finite Body in a Series of Inertial Integrals	382
205.	Laplace's Integral Equation.	384
206.	The Expansion of an Arbitrary Function in a Series of Spherical Harmonics	387
207.	The Representation of a Rational, Integral Function	390
208.	Green's Problem for the Sphere	392
209.	The Potential of a Surface Distribution of Matter on a Sphere . .	393
210.	Differentiation with Respect to Poles.	395
211.	Derivation of the Tesseral Harmonics by Polar Differentiation Problems	398
		404

CHAPTER VIII

ELLIPSOIDAL HARMONICS

212.	Introduction.	407
213.	Definition of the Elliptic Coordinates.	407
214.	Differential Relations.	409
215.	The Equation of Laplace	410
216.	The Elliptic Functions of Weierstrass.	412
217.	Spherical Harmonics in Elliptic Coordinates.	415
218.	The Inverse Problem.	416
219.	The Functions of Lamé.	418
220.	Determination of the Constant N	419
221.	Existence of Solutions for Class I.	420
222.	Existence of Solutions for Class II	422
223.	Existence of Solutions for Class III.	423
224.	Existence of Solutions for Class IV.	425
225.	The Products of Lamé	426
226.	Liouville's Proof That All of the Roots Are Real.	427

SECTION	PAGE
227. Particular Examples of Lamé's Functions	429
228. The Pattern as a Function of q_1	435
229. Parametric Representations of a Sphere.	436
230. The Ellipsoidal Harmonics as a Function of λ	438
231. The Surface Harmonic V_2V_3	439
232. The Spheroidal Surface Harmonic V_2V_3	441
233. The Roots of the Characteristic Equation Considered as Functions of t	443
234. The Characteristic Equation Has No Multiple Roots.	445
235. The Functions of Lamé Are Linearly Independent	446
236. The Expansion of An Arbitrary Function in Terms of the Ellipsoidal Harmonics.	447
237. Surface Integrals.	449
238. The Coefficients of an Expansion in Terms of Ellipsoidal Harmonics	451
239. The Roots of Lamé's Polynomials Are Real, Distinct, and Lie between a^2 and c^2	452
240. Ellipsoidal Harmonics of the Second Kind.	454
241. The Potential of an Ellipsoidal Harmonic Surface Distribution of Matter.	456
242. The Potential of an Ellipsoidal Homoeoid.	457
243. Green's Problem on an Ellipsoid Extension of the General Theory	459
244. Fundamental Functions.	460
BIBLIOGRAPHY	463
INDEX.	467