

# Contents

Introduction	IX
Basic Notations	XIII
<b>0. CLASSICAL POTENTIAL THEORY</b>	
1. Harmonic and Hyperharmonic Functions	1
2. Brownian Semigroup	7
3. Excessive Functions	10
<b>I. GENERAL PRELIMINARIES</b>	
1. Function Cones	13
2. Choquet Boundary	21
3. Analytic Sets and Capacitances	27
4. Laplace Transforms	34
5. Coercive Bilinear Forms	35
<b>II. EXCESSIVE FUNCTIONS</b>	
1. Kernels	38
2. Supermedian Functions	42
3. Semigroups and Resolvents	44
4. Balayage Spaces	55
5. Continuous Potentials	65
6. Construction of Kernels	69
7. Construction of Resolvents	76
8. Construction of Semigroups	84
<b>III. HYPERHARMONIC FUNCTIONS</b>	
1. Harmonic Kernels	93
2. Harmonic Structure of a Balayage Space	98
3. Convergence Properties	105
4. Minimum Principle and Sheaf Properties	106
5. Regularizations	110
6. Potentials	114
7. Absorbing and Finely Isolated Points	121
8. Harmonic Spaces	125
<b>IV. MARKOV PROCESSES</b>	
1. Stochastic Processes	132
2. Markov Processes	135
3. Transition Functions	138
4. Modifications	144
5. Stopping Times	151
6. Strong Markov Processes	155
7. Hunt Processes	161
8. Four Equivalent Views of Potential Theory	168

**V. EXAMPLES**

1. Subspaces	171
2. Strong Feller Kernels	175
3. Subordination by Convolution Semigroups	184
4. Riesz Potentials	191
5. Products	207
6. Heat Equation	215
7. Brownian Semigroups on the Infinite Dimensional Torus	227
8. Images	232
9. Further Examples	239

**VI. BALAYAGE THEORY**

1. Balayage of Functions	243
2. Balayage of Measures	249
3. Probabilistic Interpretation	256
4. Base	272
5. Exceptional Sets	282
6. Essential Base	296
7. Penetration Time	305
8. Fine Support of Potentials	310
9. Fine Properties of Balayage	316
10. Convergence of Balayage Measures	321
11. Accumulation Points of Balayage Measures	324
12. Extreme Representing Measures	336

**VII. DIRICHLET PROBLEM**

1. Perron Sets	341
2. Generalized Dirichlet Problem	343
3. Regular Points	348
4. Irregular Points	353
5. Simplicial Cones	357
6. Weak Dirichlet Problem	363
7. Characterization of the Generalized Solution	367
8. Fine Dirichlet Problem	369
9. Approximation	374
10. Removable Singularities	379

**VIII. PARTIAL DIFFERENTIAL EQUATIONS**

1. Bauer Spaces	383
2. Semi-Elliptic Differential Operators	387
3. Smooth Bauer Spaces	391
4. Weak Solutions	393
5. Elliptic-Parabolic Differential Operators	401
Notes	406
Bibliography	414
Index of Symbols	429
Subject Index	432
Guide to Standard Examples	435