

Contents

Preface ix

0. Introduction and Overview

0.1. Statement of Purpose	1
0.2. An Overview	2

1. The General Problems

1.1. Introduction	5
1.2. Relations between Differential and Integral Equations	7
1.3. A Glance at Initial Conditions and Existence	12
1.4. Building the Intuition	14
1.5. Reducible Equations	18

2. Linear Equations

2.1. Existence Theory	22
2.2. Linear Properties	26
2.3. Convolution and the Laplace Transform	28
2.4. Stability	33
2.5. Liapunov Functionals and Small Kernels	37
2.6. Uniform Asymptotic Stability	46
2.7. Reducible Equations Revisited	58
2.8. The Resolvent	61

3. Existence Properties

3.1. Definitions, Background, and Review	66
3.2. Existence and Uniqueness	73
3.3. Continuation of Solutions	78
3.4. Continuity of Solutions	89

4. History, Examples, and Motivation

4.0. Introduction	97
4.1. Volterra and Mathematical Biology	98
4.2. Renewal Theory	112
4.3. Examples	115

5. Instability, Stability, and Perturbations

5.1. The Matrix $A^T B + BA$	124
5.2. The Scalar Equation	133
5.3. The Vector Equation	142
5.4. Complete Instability	151

6. Stability and Boundedness

6.1. Stability Theory for Ordinary Differential Equations	155
6.2. Construction of Liapunov Functions	166
6.3. A First Integral Liapunov Functional	173
6.4. Nonlinear Considerations and an Annulus Argument	180
6.5. A Functional in the Unstable Case	194

7. Perturbations

7.1. A Converse Theorem Yielding a Perturbation Result	198
7.2. Boundedness under Perturbations	203
7.3. Additive Properties of Functionals	214

Contents

8. Functional Differential Equations

8.0. Introduction	227
8.1. Existence and Uniqueness	228
8.2. Asymptotic Stability	237
8.3. Equations with Bounded Delay	247
8.4. Boundedness with Unbounded Delay	261
8.5. Limit Sets	275
8.6. Periodic Solutions	283
8.7. Limit Sets and Unbounded Delays	294
References	303
Author Index	309
Subject Index	311