Contents

Preface v

1 A General Introduction to Minimax (H^∞-Optimal) Designs 1

1.1 A Brief History 1

1.2 A Relationship Between H^∞-Optimal Control and LQ Zero-Sum Dynamic Games 4

1.3 Discrete- and Continuous-time Models 7

1.4 Organization of the Book 9

1.5 Conventions, Notation and Terminology 10

2 Basic Elements of Static and Dynamic Games 13

2.1 Static Zero-Sum Games 13

2.2 Discrete-Time Dynamic Games 16

2.3 Continuous-Time Dynamic Games 24

3 The Discrete-Time Minimax Design Problem With Perfect State Measurements 28

3.1 Introduction 28

3.2 The Soft-Constrained Linear-Quadratic Dynamic Game . . 29

3.2.1 Open-loop information structure for both players. . 30

3.2.2 Closed-loop perfect state information for both players. 32

3.2.3 Closed-loop 1-step delay information for both players. 38

3.2.4 An illustrative example 42

3.3 Solution to the Disturbance Attenuation Problem 44
3.3.1 General closed-loop information 44
3.3.2 Illustrative example (continued) 48
3.3.3 A least favorable distribution for the disturbance . . 50
3.3.4 Optimum controller under the 1-step delay information pattern 52
3.4 The Infinite-Horizon Case 54
3.5 More General Classes of Problems 65
3.5.1 More general plants and cost functions 65
3.5.2 Nonzero Initial State 68
3.6 Extensions to Nonlinear Systems under Nonquadratic Performance Indices 69
3.7 Summary of Main Results 71

4 Continuous-Time Systems With Perfect State Measurements 72

4.1 Introduction 72
4.2 The Soft-Constrained Differential Game 73
4.2.1 Open-loop information structure for both players. . 74
4.2.2 Closed-loop perfect state information for both players. 77
4.2.3 Sampled-data information for both players. 80
4.2.4 Delayed state measurements 84
4.3 The Disturbance Attenuation Problem 86
4.3.1 Closed-loop perfect state information 87
4.3.2 Sampled state measurements 89
4.3.3 An illustrative example 90
4.3.4 Delayed state measurements 92
4.4 The Infinite-Horizon Case 93
4.4.1 The soft-constrained differential game 93
4.4.2 The disturbance attenuation problem 103
4.4.3 Illustrative example (continued) 107

4.5 More General Classes of Problems 108
4.5.1 A more general cost structure 109
4.5.2 Unknown nonzero initial state 110
4.6 Nonlinear Systems and Nonquadratic Performance Indices . 111
4.7 Main Points of the Chapter 112

5 The Continuous-Time Problem With Imperfect State Measurements 114
5.1 Formulation of the Problem 114
5.2 A Certainty Equivalence Principle, and Its Application to the Basic Problem $\mathcal{P}_\gamma$ 117
5.3 Sampled-Data Measurements 132
5.4 The Infinite-Horizon Case 138
5.5 More General Classes of Problems 144
5.5.1 Cross terms in the cost function 144
5.5.2 Delayed measurements 146
5.5.3 Nonlinear/nonquadratic problems 148
5.6 Main Results of the Chapter 149

6 The Discrete-Time Problem With Imperfect State Measurements 151
6.1 The Problem Considered 151
6.2 A Certainty Equivalence Principle and Its Application to the Basic Problem $\mathcal{P}_\gamma$ 154
6.3 The Infinite-Horizon Case 167
6.4 More General Classes of Problems 172
6.4.1 Cross terms in the performance index 172
6.4.2 Delayed measurements 174
6.4.3 The "filtering" problem 176
6.4.4 Nonlinear/nonquadratic problems 178
6.5 Main Points of the Chapter 178

7 Performance Levels For Minimax Estimators 180

7.1 Introduction 180

7.2 A Static Minimax Estimation Problem 182

7.3 Optimum Performance Levels 187

7.3.1 Discrete time 187

7.3.2 Continuous time 190

7.4 Summary of Main Results 192

8 Appendix A: Conjugate Points 194

9 Appendix B: Danskin's Theorem 208

10 References 213

11 List of Corollaries, Definitions, Lemmas, Propositions, Remarks and Theorems 221