

# Contents

## PART I. FINITE-DIMENSIONAL VECTOR SPACES AND THE FOURIER TRANSFORM

<b>1. Concepts from Complex Vector Analysis and the Fourier Transform</b>	3
1.1. <i>N</i> -Dimensional Complex Vector Spaces . . . . .	3
1.2. Inner Product and Norm in $\mathcal{V}^N$ . . . . .	5
1.3. Passive and Active Transformations . . . . .	9
1.4. Unitary Transformations: Permutations and the Fourier Transform . . . . .	13
1.5. Self-Adjoint Operators . . . . .	20
1.6. The Dihedral Group . . . . .	26
1.7. The Axes of a Transformation: Eigenvalues and Eigenvectors . . . . .	33
<b>2. The Application of Fourier Analysis to the Uncoupling of Lattices</b>	43
2.1. Mechanical and Electric Analogies . . . . .	43
2.2. The Equation of Motion of Coupled Systems and Solution . . . . .	49
2.3. Fundamental Solutions, Normal Modes, and Traveling Waves . . . . .	56
2.4. Farther-Neighbor Interaction, Molecular and Diatomic Lattices . . . . .	71
2.5. Energy in a Lattice . . . . .	85
2.6. Phase Space, Time Evolution, and Constants of Motion . . . . .	88
<b>3. Further Developments and Applications of the Finite Fourier Transform</b>	101
3.1. Convolution: Filters and Windows . . . . .	101
3.2. Correlation: Signal Detection and Noise . . . . .	114
3.3. The Fast Fourier Transform Algorithm . . . . .	125
3.4. The "Limit" $N \rightarrow \infty$ : Fourier Series and Integral Transforms . . . . .	130

**PART II. FOURIER AND BESSSEL SERIES**

<b>4. Function Vector Spaces and Fourier Series</b>	139
4.1. Notions on Function Vector Spaces	139
4.2. The Dirichlet Conditions	145
4.3. Alternative Representations, Transformations, and Symmetries	154
4.4. Differential Properties and Convergence	162
4.5. The Dirac $\delta$ and Divergent Series	174
4.6. Linear Operators, Infinite Matrices, and Integral Kernels	184
4.7. Fourier Series for Any Period and the Infinite-Period Limit	191
<b>5. Fourier Series in Diffusion and Wave Phenomena</b>	195
5.1. Heat Diffusion in a Ring	195
5.2. The Vibrating String	201
5.3. The Infinite Lattice	214
<b>6. Normal Mode Expansion and Bessel Series</b>	221
6.1. Eigenfunctions of the Laplacian on Finite Regions: The Rectangular Membrane	221
6.2. Laplacian on the Unit Disk: The Circular Membrane	229
6.3. Sectorial and Annular Membranes	239
6.4. Other Series of Orthonormal Functions	246

**PART III. FOURIER AND RELATED INTEGRAL TRANSFORMS**

<b>7. Fourier Transforms</b>	255
7.1. The Fourier Integral Theorem	255
7.2. Various Operators and Operations under Fourier Transformation	265
7.3. The Dirac $\delta$ and the Green's Function for a System	279
7.4. Causality and Dispersion Relations	293
7.5. Oscillator Wave Functions	307
7.6. Uncertainty Relations	328
<b>8. Integral Transforms Related to the Fourier Transform</b>	333
8.1. Laplace Transforms	334
8.2. Mellin Transforms	344
8.3. $N$ -Dimensional Fourier Transforms	352
8.4. Hankel Transforms	361
8.5. Other Integral Transforms	372

**PART IV. CANONICAL TRANSFORMS**

<b>9. Construction and Properties of Canonical Transforms</b>	381
9.1. Real Linear Canonical Transforms	381
9.2. Complex Linear Transforms and Bargmann Space	393
9.3. Canonical Transforms by Hyperdifferential Operators	405

<b>10. Applications to the Study of Differential Equations . . . . .</b>	<b>417</b>
<b>10.1. The Diffusion Equation: Similarity Group and Separating Coordinates . . . . .</b>	<b>417</b>
<b>10.2. Inhomogeneous Linear Canonical Transforms and Parabolic Equations . . . . .</b>	<b>434</b>
<b>Appendix A. The Gamma Function . . . . .</b>	<b>445</b>
<b>Appendix B. The Bessel and Related Functions . . . . .</b>	<b>449</b>
<b>Appendix C. Some Summation Formulas . . . . .</b>	<b>457</b>
<b>REFERENCES . . . . .</b>	<b>459</b>
<b>NOTATION . . . . .</b>	<b>469</b>
<b>INDEX . . . . .</b>	<b>473</b>