CONTENTS

CHAPTER I. THE OCCURRENCE OF MIXED BOUNDARY VALUE PROBLEMS IN POTENTIAL THEORY

1.1. Electrostatic Problems 1
1.2. Steady-State Diffusion Problems 7
1.3. Elastostatic Problems 10
1.4. Hydrodynamic Problems 20
1.5. The Basic Elementary Problems 23
1.6. Generalized Axisymmetric Potential Theory 24

CHAPTER II. MATHEMATICAL PRELIMINARIES

2.1. Integrals involving Bessel Functions 26
2.2. Infinite Series involving Bessel Functions 33
2.3. Some Remarks on Integral Equations 40
2.4. Operators of Fractional Integration 46
2.5. Relations between the Operator of Hankel Transforms and the Erdélyi-Kober Operators 52
2.6. Jacobi Polynomials and Associated Legendre Functions 54

CHAPTER III. THE FIRST BASIC PROBLEM: THE PROBLEM OF THE ELECTRIFIED DISK

3.1. Weber's Solution for a Disk Charged to Unit Potential 63
3.2. Beltrami's Symmetric Potentials 64
3.3. Use of Oblate Spheroidal Coordinates 66
3.4. Copson's Solution 69
3.5. Elementary Solution of the Dual Integral Equations of Beltrami's Method 74
3.6. Methods based on the Integral Representation of Harmonic Functions 77

CHAPTER IV. DUAL INTEGRAL EQUATIONS

4.1. Introduction 80
4.2. Dual Integral Equations of Titchmarsh Type 84
4.2.1. Peters' Solution 84
4.2.2. Titchmarsh's Solution 86
4.2.3. Noble's Solution 88
4.2.4. Gordon-Copson Solution 91
4.3. Functions derived from Solutions of Dual Integral Equations 93
4.4. Special Cases corresponding to Axisymmetric Problems in Potential Theory 96
4.5. Dual Integral Equations with Trigonometrical Kernels 98
4.6. Dual Integral Equations with Hankel Kernel and Arbitrary Weight Function 106
4.6.1. Reduction to a Fredholm Equation 106
4.6.2. Reduction to a System of Algebraic Equations 113
4.6.3. Integral Equation for $\chi_1(x)$ 115
4.7. The General Problem 118
4.7.1. Reduction to the Solution of two Integral Equations 119
4.7.2. The Multiplying Factor Method 120
4.7.3. The Integral Representation Method 121
4.7.4. Identification of the Operators 122
4.8. Approximate Solution of Dual Integral Equations 123
4.9. Simultaneous Dual Integral Equations 129

CHAPTER V. DUAL SERIES EQUATIONS

5.1. Introduction 134
5.2. Dual Relations involving Fourier-Bessel Series 135
5.2.1. The Reduction of Problem (*a*) to the Solution of a System of Algebraic Equations 136
5.2.2. The Reduction of Problem (*a*) to the Solution of an Integral Equation 139
5.2.3. The Reduction of Problem (*b*) to the Solution of an Integral Equation 142
5.3. Dual Relations involving Dini Series 144
5.4. Dual Relations involving Trigonometric Series 150
5.4.1. Dual Relations involving Fourier Sine Series 152
5.4.2. Dual Relations involving Sine Series analogous to Dini Series 158
5.4.3. Dual Relations involving Fourier Cosine Series 161
5.4.4. Dual Relations involving Cosine Series analogous to Fourier-Bessel Series 162
5.4.5 Tranter's Formulae 163
5.5. Dual Relations involving Series of Jacobi Polynomials 165
5.6. Dual Relations involving Series of Associated Legendre Functions 173

CHAPTER VI. TRIPLE RELATIONS

6.1. The Origin of Triple Relations 178
6.2. Triple Integral Equations of Titchmarsh Type 180
6.3. Triple Equations corresponding to Axisymmetric Potential Functions 184
6.4. Reduction of Triple Integral Equations to Dual Series Relations 187
6.5. Triple Equations involving Series of Legendre Polynomials 190
6.5.1. Solution of Equations of the First Kind 192
6.5.2. Solution of Equations of the Second Kind 196

CHAPTER VII. METHODS BASED ON INTEGRAL REPRESENTATIONS OF HARMONIC FUNCTIONS

7.1. Kobayashi Potentials 199
7.2. Dovnorovich's Solution of the First Basic Problem 201
7.3. Galin's Theorem 203
7.4. A Simple Superposition Method 207
7.5. Green's Solution of the First Basic Problem and Related Solutions 209
7.6. Boundary Value Problems concerning a Spherical Cap 215
7.7. Boundary Value Problems concerning two Spherical Caps 219
7.8. The Potential due to a Circular Annulus 225

CHAPTER VIII. APPLICCATIONS TO ELECTROSTATICS

8.1. The Circular Plate Condenser 230
8.1.1. Derivation of Love's Integral Equation 230
8.1.2. Solution of the Integral Equation 234
8.1.3. Approximate Solutions 238
8.2. Electrified Disk between Earthed Parallel Plates 246
8.3. Electrified Disk within an Earthed Cylinder 253
8.4. Two Coplanar Electrified Disks 259
8.5. Two Parallel Electrified Strips 264
8.6. Field due to a Charged Annular Disk 267
8.7. Problems concerning Spherical Caps 270
8.7.1. Spherical Cap at Constant Potential 270
8.7.2. Spherical Cap in a Uniform Field 272

APPENDIX

Table of Relations involving the Erdélyi-Kober Operators and the Modified Operator of Hankel Transforms 274

REFERENCES 275

SUBJECT INDEX 279

AUTHORS' INDEX 281

INDEX OF SYMBOLS 283