Contents

Preliminaries

1 *Harmonic Functions*

1 Green's Identity 6
2 Fundamental Harmonic Functions 7
3 The Averaging Principle 9
4 The Poisson Integral Formula 13
5 The Maximum Principle 18
6 Smoothing of Functions 19

2 *Functions Harmonic on a Ball*

1 The Classical Dirichlet Problem 21
2 Existence of a Solution for a Ball 22
3 Functions Harmonic on a Ball 24
4 Harnack's Inequality 29
5 Directed Families of Harmonic Functions 32
6 The Kelvin Transformation 36

3 *Boundary Limit Theorems*

1 The Radial Limit Theorem 43
2 Differentiation 47
3 Nontangential Limits 54

4 *Superharmonic Functions*

1 Definitions of Superharmonic Functions 57
2 Generalized Laplacian 63
3 Properties of Superharmonic Functions 65
4 Approximation of Superharmonic Functions 70

5 *Green Functions*
1 Green Function for the Ball 77
2 Green Functions for Open Regions 84
3 Harmonic Minorants 91

6 *Green Potentials*
1 Definitions and Properties 98
2 Total Sets of Continuous Functions 105
3 Total Sets of Continuous Potentials 108
4 Riesz Decomposition of Superharmonic Functions 111
5 Continuity Properties of Potentials 117
6 Boundary Behavior of Potentials 120
7 Poisson's Equation 122

7 *Capacity*
1 Polar Sets 126
2 Superharmonic Extensions 130
3 Balayage 134
4 Capacity and Capacitary Potential 137
5 Choquet Capacity 141
6 Applications 150

8 *The Generalized Dirichlet Problem*
1 The Perron-Wiener-Brelot Method 156
2 Harmonic Measure 162
3 Boundary Behavior of the PWB Solution 168
4 Boundary Behavior of the Green Function 176
5 Applications 181

9 *Dirichlet Problem for Unbounded Regions*
1 Exterior Dirichlet Problem 186
2 Dirichlet Problem for Unbounded Regions 191
3 Boundary Behavior 198
4 Applications 205

10 *Fine Topology*
1 An Intrinsic Topology 207
2 Thin Sets 208
3 Thinness and Regularity 212
4 Fine Limits 216
5 Wiener's Test 218

11 *Energy*
1 Energy Principle 223
2 Mutual Energy 228
3 Balayage of Measures 234
4 Capacity and Energy 237

12 *Martin Boundary*
1 Martin Boundary 240
2 The Martin Representation 249
3 The Dirichlet Problem for R_M^* 261

Notes and Comments 267

Bibliography 275

Index of Notation 279

Subject Index 281