

# Contents

<b>Notation and Terminology</b> .....	xiii
<b>1. Harmonic Functions</b>	
1.1. Laplace's equation .....	1
1.2. The mean value property .....	3
1.3. The Poisson integral for a ball .....	6
1.4. Harnack's inequalities .....	13
1.5. Families of harmonic functions: convergence properties .....	15
1.6. The Kelvin transform .....	19
1.7. Harmonic functions on half-spaces .....	22
1.8. Real-analyticity of harmonic functions .....	26
1.9. Exercises .....	30
<b>2. Harmonic Polynomials</b>	
2.1. Spaces of homogeneous polynomials .....	33
2.2. Another inner product on a space of polynomials .....	35
2.3. Axially symmetric harmonic polynomials .....	37
2.4. Polynomial expansions of harmonic functions .....	40
2.5. Laurent expansions of harmonic functions .....	44
2.6. Harmonic approximation .....	47
2.7. Harmonic polynomials and classical polynomials .....	53
2.8. Exercises .....	55
<b>3. Subharmonic Functions</b>	
3.1. Elementary properties .....	59
3.2. Criteria for subharmonicity .....	64
3.3. Approximation of subharmonic functions by smooth ones ..	68
3.4. Convexity and subharmonicity .....	72
3.5. Mean values and subharmonicity .....	75
3.6. Harmonic majorants .....	79
3.7. Families of subharmonic functions: convergence properties ..	82
3.8. Exercises .....	84

**4. Potentials**

4.1.	Green functions .....	89
4.2.	Potentials .....	96
4.3.	The distributional Laplacian .....	100
4.4.	The Riesz decomposition .....	105
4.5.	Continuity and smoothness properties .....	109
4.6.	Classical boundary limit theorems .....	112
4.7.	Exercises .....	118

**5. Polar Sets and Capacity**

5.1.	Polar sets .....	123
5.2.	Removable singularity theorems .....	127
5.3.	Reduced functions .....	129
5.4.	The capacity of a compact set .....	134
5.5.	Inner and outer capacity .....	137
5.6.	Capacitable sets .....	143
5.7.	The fundamental convergence theorem .....	146
5.8.	Logarithmic capacity .....	150
5.9.	Hausdorff measure and capacity .....	156
5.10.	Exercises .....	159

**6. The Dirichlet Problem**

6.1.	Introduction .....	163
6.2.	Upper and lower PWB solutions .....	164
6.3.	Further properties of PWB solutions .....	167
6.4.	Harmonic measure .....	172
6.5.	Negligible sets .....	177
6.6.	Boundary behaviour .....	179
6.7.	Behaviour near infinity .....	188
6.8.	Regularity and the Green function .....	189
6.9.	PWB solutions and reduced functions .....	191
6.10.	Superharmonic extension .....	192
6.11.	Exercises .....	193

**7. The Fine Topology**

7.1.	Introduction .....	197
7.2.	Thin sets .....	199
7.3.	Thin sets and reduced functions .....	201
7.4.	Fine limits .....	206
7.5.	Thin sets and the Dirichlet problem .....	208
7.6.	Thinness at infinity .....	214
7.7.	Wiener's criterion .....	217
7.8.	Limit properties of superharmonic functions .....	221
7.9.	Harmonic approximation .....	226

<b>8. The Martin Boundary</b>	
8.1. The Martin kernel and Martin boundary .....	233
8.2. Reduced functions and minimal harmonic functions .....	241
8.3. Reductions on $\Delta_0$ and $\Delta_1$ .....	246
8.4. The Martin representation .....	250
8.5. The Martin boundary of a strip .....	252
8.6. The Martin kernel and the Kelvin transform .....	256
8.7. The boundary Harnack principle for Lipschitz domains .....	259
8.8. The Martin boundary of a Lipschitz domain .....	269
<b>9. Boundary Limits</b>	
9.1. Swept measures and the Dirichlet problem for the Martin compactification .....	273
9.2. Minimal thinness .....	279
9.3. Minimal fine limits .....	284
9.4. The Fatou–Naim–Doob theorem .....	287
9.5. Minimal thinness in subdomains .....	290
9.6. Refinements of limit theorems .....	294
9.7. Minimal thinness in a half-space .....	296
<b>Appendix</b> .....	305
<b>Historical Notes</b> .....	309
<b>References</b> .....	317
<b>Symbol Index</b> .....	329
<b>Index</b> .....	331