

# Contents.

## Chapter I.

### The Force of Gravity.

1. The Subject Matter of Potential Theory . . . . .	1
2. Newton's Law . . . . .	2
3. Interpretation of Newton's Law for Continuously Distributed Bodies . . . . .	3
4. Forces Due to Special Bodies . . . . .	4
5. Material Curves, or Wires . . . . .	8
6. Material Surfaces or Laminas . . . . .	10
7. Curved Laminas . . . . .	12
8. Ordinary Bodies, or Volume Distributions . . . . .	15
9. The Force at Points of the Attracting Masses . . . . .	17
10. Legitimacy of the Amplified Statement of Newton's Law; Attraction between Bodies . . . . .	22
11. Presence of the Couple; Centrobaric Bodies; Specific Force . . . . .	26

## Chapter II.

### Fields of Force.

1. Fields of Force and Other Vector Fields . . . . .	28
2. Lines of Force . . . . .	28
3. Velocity Fields . . . . .	31
4. Expansion, or Divergence of a Field . . . . .	34
5. The Divergence Theorem . . . . .	37
6. Flux of Force; Solenoidal Fields . . . . .	40
7. Gauss' Integral . . . . .	42
8. Sources and Sinks . . . . .	44
9. General Flows of Fluids; Equation of Continuity . . . . .	45

## Chapter III.

### The Potential.

1. Work and Potential Energy . . . . .	48
2. Equipotential Surfaces . . . . .	54
3. Potentials of Special Distributions . . . . .	55
4. The Potential of a Homogeneous Circumference . . . . .	58
5. Two Dimensional Problems; The Logarithmic Potential . . . . .	62
6. Magnetic Particles . . . . .	65
7. Magnetic Shells, or Double Distributions . . . . .	66
8. Irrotational Flow . . . . .	69
9. Stokes' Theorem . . . . .	72
10. Flow of Heat . . . . .	76
11. The Energy of Distributions . . . . .	79
12. Reciprocity; Gauss' Theorem of the Arithmetic Mean . . . . .	82

## Chapter IV.

**The Divergence Theorem.**

1. Purpose of the Chapter . . . . .	84
2. The Divergence Theorem for Normal Regions . . . . .	85
3. First Extension Principle . . . . .	88
4. Stokes' Theorem . . . . .	89
5. Sets of Points . . . . .	91
6. The Heine-Borel Theorem . . . . .	94
7. Functions of One Variable; Regular Curves . . . . .	97
8. Functions of Two Variables; Regular Surfaces . . . . .	100
9. Functions of Three Variables . . . . .	113
10. Second Extension Principle; The Divergence Theorem for Regular Regions . . . . .	113
11. Lightening of the Requirements with Respect to the Field. . . . .	119
12. Stokes' Theorem for Regular Surfaces . . . . .	121

## Chapter V.

**Properties of Newtonian Potentials at Points of Free Space.**

1. Derivatives; Laplace's Equation . . . . .	121
2. Development of Potentials in Series . . . . .	124
3. Legendre Polynomials . . . . .	125
4. Analytic Character of Newtonian Potentials. . . . .	135
5. Spherical Harmonics . . . . .	139
6. Development in Series of Spherical Harmonics . . . . .	141
7. Development Valid at Great Distances . . . . .	143
8. Behavior of Newtonian Potentials at Great Distances . . . . .	144

## Chapter VI.

**Properties of Newtonian Potentials at Points Occupied by Masses.**

1. Character of the Problem . . . . .	146
2. Lemmas on Improper Integrals . . . . .	146
3. The Potentials of Volume Distributions . . . . .	150
4. Lemmas on Surfaces . . . . .	157
5. The Potentials of Surface Distributions. . . . .	160
6. The Potentials of Double Distributions . . . . .	166
7. The Discontinuities of Logarithmic Potentials . . . . .	172

## Chapter VII.

**Potentials as Solutions of Laplace's Equation; Electrostatics.**

1. Electrostatics in Homogeneous Media . . . . .	175
2. The Electrostatic Problem for a Spherical Conductor . . . . .	176
3. General Coördinates . . . . .	178
4. Ellipsoidal Coördinates . . . . .	184
5. The Conductor Problem for the Ellipsoid. . . . .	188
6. The Potential of the Solid Homogeneous Ellipsoid . . . . .	192
7. Remarks on the Analytic Continuation of Potentials . . . . .	196
8. Further Examples Leading to Solutions of Laplace's Equation . . . . .	198
9. Electrostatics; Non-homogeneous Media . . . . .	206

## Chapter VIII.

**Harmonic Functions.**

1. Theorems of Uniqueness . . . . .	211
2. Relations on the Boundary between Pairs of Harmonic Functions . . . . .	215

3. Infinite Regions . . . . .	216
4. Any Harmonic Function is a Newtonian Potential . . . . .	218
5. Uniqueness of Distributions Producing a Potential . . . . .	220
6. Further Consequences of Green's Third Identity . . . . .	223
7. The Converse of Gauss' Theorem . . . . .	224

## Chapter IX.

**Electric Images; Green's Function.**

1. Electric Images . . . . .	228
2. Inversion; Kelvin Transformations . . . . .	231
3. Green's Function . . . . .	236
4. Poisson's Integral; Existence Theorem for the Sphere . . . . .	240
5. Other Existence Theorems . . . . .	244

## Chapter X.

**Sequences of Harmonic Functions.**

1. Harnack's First Theorem on Convergence . . . . .	248
2. Expansions in Spherical Harmonics . . . . .	251
3. Series of Zonal Harmonics . . . . .	254
4. Convergence on the Surface of the Sphere . . . . .	256
5. The Continuation of Harmonic Functions . . . . .	259
6. Harnack's Inequality and Second Convergence Theorem . . . . .	262
7. Further Convergence Theorems . . . . .	264
8. Isolated Singularities of Harmonic Functions . . . . .	268
9. Equipotential Surfaces . . . . .	273

## Chapter XI.

**Fundamental Existence Theorems.**

1. Historical Introduction . . . . .	277
2. Formulation of the Dirichlet and Neumann Problems in Terms of Integral Equations . . . . .	286
3. Solution of Integral Equations for Small Values of the Parameter . . . . .	287
4. The Resolvent . . . . .	289
5. The Quotient Form for the Resolvent . . . . .	290
6. Linear Dependence; Orthogonal and Biorthogonal Sets of Functions . . . . .	292
7. The Homogeneous Integral Equations . . . . .	294
8. The Non-homogeneous Integral Equation; Summary of Results for Continuous Kernels . . . . .	297
9. Preliminary Study of the Kernel of Potential Theory . . . . .	299
10. The Integral Equation with Discontinuous Kernel . . . . .	307
11. The Characteristic Numbers of the Special Kernel . . . . .	309
12. Solution of the Boundary Value Problems . . . . .	311
13. Further Consideration of the Dirichlet Problem; Superharmonic and Subharmonic Functions . . . . .	315
14. Approximation to a Given Domain by the Domains of a Nested Sequence . . . . .	317
15. The Construction of a Sequence Defining the Solution of the Dirichlet Problem . . . . .	322
16. Extensions; Further Properties of $U$ . . . . .	323
17. Barriers . . . . .	326
18. The Construction of Barriers . . . . .	328
19. Capacity . . . . .	330
20. Exceptional Points . . . . .	334

Chapter XII.  
The Logarithmic Potential.

1. The Relation of Logarithmic to Newtonian Potentials . . . . .	338
2. Analytic Functions of a Complex Variable . . . . .	340
3. The Cauchy-Riemann Differential Equations . . . . .	341
4. Geometric Significance of the Existence of the Derivative . . . . .	343
5. Cauchy's Integral Theorem . . . . .	344
6. Cauchy's Integral. . . . .	348
7. The Continuation of Analytic Functions . . . . .	351
8. Developments in Fourier Series . . . . .	353
9. The Convergence of Fourier Series . . . . .	355
10. Conformal Mapping . . . . .	359
11. Green's Function for Regions of the Plane . . . . .	363
12. Green's Function and Conformal Mapping . . . . .	365
13. The Mapping of Polygons . . . . .	370
<i>Bibliographical Notes</i> . . . . .	377
<i>Index</i> . . . . .	379