Contents

Notat.	1011	1X
Prefa	ce	x
Intro	Introduction	
Part I. ONE-DIMENSIONAL DYNAMICAL SYSTEMS		15
1. 1	Introduction to the Theory of Dynamical Systems	15
§ 1.	Are One-Dimensional Dynamical Systems Simple?	17
§2.	What May Occur in One-Dimensional Dynamical Systems.	
	Some Notions and Examples	23
§ 3.	Intermixing (Strange) Attractors	37
2. 1	Periodic Trajectories	45
§1.	Attracting Fixed Points	45
§2.	Coexistence of Cycles	52
§ 3	Bifurcations of Cycles	63
3. 1	Behavior of Trajectories	71
§1.	Trajectories of Simple Dynamical Systems	71
§ 2.	Return of Points and Sets	75
§ 3.	Criteria of Simplicity and Complexity for Maps	81
§ 4	Stability of Trajectories and Dynamical Systems	84
4. 1	Dynamical Systems for U-Maps	95
§ 1.	Unimodal Maps	95
§2 .	Schwarzian and Attracting Cycles	98
§3.	Periodic Intervals	101
§4 .	Spectral Decomposition of the Set of Non-Wandering Points	111
§5.	Bifurcations of the Periodic Intervals and Stability	
	of the Spectral Decomposition	119

Part	II. DIFFERENCE EQUATIONS WITH	
	CONTINUOUS TIME	125
1. Nonlinear Difference Equations		125
§ 1	Statement of the Problem	125
§2.	Asymptotically Discontinuous Solutions	134
§3.		15.
	Asymptotically Discontinuous Solutions	139
§4.	· · ·	143
§5.		- 1-
	Discontinuous Solutions	152
§ 6.	Stability of Asymptotically Discontinuous Solutions	155
2.	Difference Equations with U-Nonlinearity	159
§ 1.	1, 1, -1	159
§2 .	Spectrum of Asymptotic Jumps. Solutions of	
	Relaxation and Turbulent Types	163
§ 3.	•	175
§4 .	Emergence of Ordered Structures	183
Part 1	III. DIFFERENTIAL-DIFFERENCE EQUATIONS	187
1.	Completely Integrable Differential-Difference Equations	188
§ 1.	What Kind of Asymptotic Behavior of Solutions of	
	Differential-Difference Equations One May Expect	188
§2.	r r r r r r r r r r r r r r r r r r r	
	Integrable Differential-Difference Equations	193
§3 .	Connection Between Solutions of Completely Integrable	
	Differential-Difference Equations and Solutions of the	
	Corresponding Difference Equations. The Phase Plane Method	197
§4.	ryg-word 2 miletonician	
	Difference Equations. Their Exceptional Character	206
§ 5.	Asymptotically Periodic Solutions of Completely Integrable	
	Equations. Their Typicalness	211
2.	Differential-Difference Equations Close To Difference Ones	223
§ 1.	General Definitions and Properties	223
§2 .	Asymptotic Behavior of Solutions of the Perturbed Equation	227
§3.	Stability of Solutions	234

		Contents	vii
	3.	Singularly Perturbed Differential-Difference Equations	239
	§1.	Statement of the Problem. Continuous Dependence of	
		Solutions on the Parameter v on a Finite Interval	239
	§2.	Invariance of the Asymptotic Properties of Solutions	243
	§3.	Behavior of Solutions as $t \to \infty$	248
Pa	rt l	IV. BOUNDARY-VALUE PROBLEMS FOR HYPERBOLIC SYSTEMS OF PARTIAL	
		DIFFERENTIAL EQUATIONS	273
	1.	Reduction of Boundary-Value Problems to Difference and	
		Differential-Difference Equations	274
	§1.	Reduction to a Nonlinear Difference Equation	274
	§2.	Reduction to Differential-Difference Equations	277
	§3.	Reduction Procedure in More General Situations	279
	2.	Boundary-Value Problem for a System with Small Parameter	285
	§1.	Boundary Value Problem for a Non-Perturbed System	285
	§2.	The Case of $\varepsilon > 0$. Existence of Solution and Extendability	290
	§3.		292
	§4.	Stability in the Skorokhod Metric and Asymptotic Periodicity	297
	3.	Boundary-Value Problem for Systems with Two Spatial Variables	305
	§1.	Statement of the problem, Its Correctness. Reduction to a	
		Difference Equation	305
	§2.	· · · · · · · · · · · · · · · · · · ·	210
		Linear Boundary Conditions	310
	§3.	•	313
	§4.	• •	318
	§5	Self-Stochasticity	330
References			335

Index

357