

Contents of Volume II

Preface	vii
Contents of Volume I	xiii
CHAPTER XI	
Introduction	1
§0. Introduction	1
§1. Equations with Symmetry	2
§2. Techniques	9
§3. Mode Interactions	18
§4. Overview	21
CHAPTER XII	
Group-Theoretic Preliminaries	23
§0. Introduction	23
§1. Group Theory	25
§2. Irreducibility	33
§3. Commuting Linear Mappings and Absolute Irreducibility	39
§4. Invariant Functions	43
§5. Nonlinear Commuting Mappings	49
§6.* Proofs of Theorems in §§4 and 5	54
§7.* Tori	63
CHAPTER XIII	
Symmetry-Breaking in Steady-State Bifurcation	67
§0. Introduction	67
§1. Orbits and Isotropy Subgroups	69
§2. Fixed-Point Subspaces and the Trace Formula	73
§3. The Equivariant Branching Lemma	80
§4. Orbital Asymptotic Stability	86

§5. Bifurcation Diagrams and D_n Symmetry	95
§6.† Subgroups of $SO(3)$	103
§7.† Representations of $SO(3)$ and $O(3)$: Spherical Harmonics	107
§8.† Symmetry-Breaking from $SO(3)$	115
§9.† Symmetry-Breaking from $O(3)$	119
§10.* Generic Spontaneous Symmetry-Breaking	132
CASE STUDY 4	
The Planar Bénard Problem	141
§0. Introduction	141
§1. Discussion of the PDE	148
§2. One-Dimensional Fixed-Point Subspaces	153
§3. Bifurcation Diagrams and Asymptotic Stability	156
CHAPTER XIV	
Equivariant Normal Forms	164
§0. Introduction	164
§1. The Recognition Problem	165
§2.* Proof of Theorem 1.3	172
§3. Sample Computations of $RT(h, \Gamma)$	176
§4. Sample Recognition Problems	185
§5. Linearized Stability and Γ -equivalence	199
§6. Intrinsic Ideals and Intrinsic Submodules	202
§7. Higher Order Terms	204
CHAPTER XV	
Equivariant Unfolding Theory	208
§0. Introduction	208
§1. Basic Definitions	209
§2. The Equivariant Universal Unfolding Theorem	210
§3. Sample Universal Γ -unfoldings	213
§4. Bifurcation with D_3 Symmetry	218
§5.† The Spherical Bénard Problem	223
§6.† Spherical Harmonics of Order 2	227
§7.* Proof of the Equivariant Universal Unfolding Theorem	233
§8.* The Equivariant Preparation Theorem	243
CASE STUDY 5	
The Traction Problem for Mooney–Rivlin Material	247
§0. Introduction	247
§1. Reduction to D_3 Symmetry in the Plane	250
§2. Taylor Coefficients in the Bifurcation Equation	251
§3. Bifurcations of the Rivlin Cube	255
CHAPTER XVI	
Symmetry-Breaking in Hopf Bifurcation	258
§0. Introduction	258
§1. Conditions for Imaginary Eigenvalues	261

§2. A Simple Hopf Theorem with Symmetry	267
§3. The Circle Group Action	269
§4. The Hopf Theorem with Symmetry	275
§5. Birkhoff Normal Form and Symmetry	284
§6. Floquet Theory and Asymptotic Stability	293
§7. Isotropy Subgroups of $\Gamma \times S^1$	299
§8.* Dimensions of Fixed-Point Subspaces	302
§9. Invariant Theory for $\Gamma \times S^1$	308
§10. Relationship Between Liapunov–Schmidt Reduction and Birkhoff Normal Form	310
§11.* Stability in Truncated Birkhoff Normal Form	313

CHAPTER XVII

Hopf Bifurcation with $O(2)$ Symmetry 324

§0. Introduction	324
§1. The Action of $O(2) \times S^1$	325
§2. Invariant Theory for $O(2) \times S^1$	331
§3. The Branching Equations	332
§4. Amplitude Equations, D_4 Symmetry, and Stability	333
§5.† Hopf Bifurcation with $O(n)$ Symmetry	338
§6.† Bifurcation with D_4 Symmetry	341
§7. The Bifurcation Diagrams	348
§8.† Rotating Waves and $SO(2)$ or Z_n Symmetry	359

CHAPTER XVIII

Further Examples of Hopf Bifurcation with Symmetry 363

§0. Introduction	363
§1. The Action of $D_n \times S^1$	367
§2. Invariant Theory for $D_n \times S^1$	372
§3. Branching and Stability for D_n	376
§4. Oscillations of Identical Cells Coupled in a Ring	387
§5.† Hopf Bifurcation with $O(3)$ Symmetry	399
§6.† Hopf Bifurcation on the Hexagonal Lattice	403

CHAPTER XIX

Mode Interactions 412

§0. Introduction	412
§1. Hopf/Steady-State Interaction	416
§2. Bifurcation Problems with Z_2 Symmetry	417
§3. Bifurcation Diagrams with Z_2 Symmetry	425
§4. Hopf/Hopf Interaction	442

CHAPTER XX

Mode Interactions with $O(2)$ Symmetry 446

§0. Introduction	446
§1.† Steady-State Mode Interaction	448
§2. Hopf/Steady-State Mode Interaction	458
§3.† Hopf/Hopf Mode Interaction	475

CASE STUDY 6	
The Taylor–Couette System	485
§0. Introduction	485
§1. Detailed Overview	486
§2. The Bifurcation Theory Analysis	498
§3. Finite Length Effects	509
Bibliography	513
Index	527