

# Contents

<b>Preface</b>	<b>ix</b>
<b>Introduction</b>	<b>1</b>
<b>1 Gradient Systems in a Lie Algebra</b>	<b>5</b>
1.1 Preliminaries . . . . .	5
1.2 Gradient systems in $F_n$ and $\text{Der}(R^n)$ . . . . .	11
1.3 Gradient Systems Determined by a Lie Algebra . . . . .	17
<b>2 Representation of a Gradient System</b>	<b>25</b>
2.1 Finite-Dimensional Lie Algebra . . . . .	25
2.2 The Maximal Rank Lie Algebra . . . . .	33
2.3 Integral Manifolds . . . . .	36
2.4 Some applications . . . . .	40
<b>3 F. G. O. Lie Algebras</b>	<b>49</b>
3.1 Lie algebras finitely generated over orbits . . . . .	49
3.2 Nonsingularity of the gradient system . . . . .	55
3.3 Some Applications . . . . .	69
<b>4 Applications</b>	<b>77</b>
4.1 Systems of Semilinear Equations . . . . .	77
4.2 Stochastic Differential Equations . . . . .	83
4.3 Systems of Hyperbolic equations . . . . .	91
4.4 Finite-Dimensional Nonlinear Filters . . . . .	97
4.5 Affine Control Systems . . . . .	108
4.6 Integral Representation of Solutions . . . . .	109
4.7 Decomposition of affine control systems . . . . .	113
<b>5 Stabilization and Related Problems</b>	<b>117</b>
5.1 Equivalent Controllable Systems . . . . .	117
5.2 Approximations, Small Controls . . . . .	128

5.3	Nonlinear Control Systems . . . . .	144
5.4	Stabilization of Affine Control Systems . . . . .	147
5.5	Controlled Invariant Lie Algebras . . . . .	158
5.6	Stochastic differential equations . . . . .	175
5.6.1	Singularly perturbed equations viewed as controlled equations . . . . .	176
5.6.2	Bounded solutions for singularly perturbed system $(\alpha), (\beta)$ . . . . .	179
5.6.3	Characterization of the dynamical limits . . . . .	182
<b>Appendix</b>		<b>197</b>
<b>Bibliography</b>		<b>235</b>
<b>Subject Index</b>		<b>237</b>