
Contents

<i>Preface</i>	<i>page xi</i>
1. Introduction	1
Exercises	15
2. Abstract Linear Equations	17
The Kernel and Image	18
Duality	20
Compactness	27
Fredholm Operators	32
Hilbert Spaces	38
Coercivity	42
Elementary Spectral Theory	45
Exercises	52
3. Sobolev Spaces	57
Convolution	58
Differentiation	61
Schwartz Distributions	64
Fourier Transforms	69
Sobolev Spaces – First Definition	73
Sobolev Spaces – Second Definition	75
Equivalence of the Norms	79
Localisation and Changes of Coordinates	83
Density and Imbedding Theorems	85
Lipschitz Domains	89
Sobolev Spaces on the Boundary	96
The Trace Operator	100
Vector-Valued Functions	106
Exercises	107

4. Strongly Elliptic Systems	113
The First and Second Green Identities	113
Strongly Elliptic Operators	118
Boundary Value Problems	128
Regularity of Solutions	133
The Transmission Property	141
Estimates for the Steklov–Poincaré Operator	145
Exercises	156
5. Homogeneous Distributions	158
Finite-Part Integrals	159
Extension from $\mathbb{R}^n \setminus \{0\}$ to \mathbb{R}^n	166
Fourier Transforms	169
Change of Variables	174
Finite-Part Integrals on Surfaces	181
Exercises	187
6. Surface Potentials	191
Parametrics	192
Fundamental Solutions	197
The Third Green Identity	200
Jump Relations and Mapping Properties	202
Duality Relations	211
Exercises	215
7. Boundary Integral Equations	217
Operators on the Boundary	217
Integral Representations	219
The Dirichlet Problem	226
The Neumann Problem	229
Mixed Boundary Conditions	231
Exterior Problems	234
Regularity Theory	239
Exercises	241
8. The Laplace Equation	246
Fundamental Solutions	247
Spherical Harmonics	250
Behaviour at Infinity	258
Solvability for the Dirichlet Problem	260
Solvability for the Neumann Problem	266
Exercises	268

9. The Helmholtz Equation	276
Separation of Variables	277
The Sommerfeld Radiation Condition	280
Uniqueness and Existence of Solutions	286
A Boundary Integral Identity	289
Exercises	293
10. Linear Elasticity	296
Korn's Inequality	297
Fundamental Solutions	299
Uniqueness Results	301
Exercises	305
Appendix A. Extension Operators for Sobolev Spaces	309
Exercises	315
Appendix B. Interpolation Spaces	317
The K -Method	318
The J -Method	321
Interpolation of Sobolev Spaces	329
Exercises	333
Appendix C. Further Properties of Spherical Harmonics	334
Exercises	338
<i>References</i>	341
<i>Index</i>	347
<i>Index of Notation</i>	353