

CONTENTS

Preface		ix
0. Introduction		1
0.1. Concept of Generalized Solution		1
0.2. Functional Spaces		3
0.3. Operators Associated with Boundary-Value Problems in the Complete Scale of Sobolev-Type Spaces		6
0.4. Theorem on Complete Collection of Isomorphisms		9
0.5. Weak Solutions of Elliptic Problems		11
0.6. Traces of Generalized Solutions on the Boundary of the Domain		12
0.7. Local Increase in Smoothness of Generalized Solutions. Green's Function		14
0.8. Elliptic Problems with Power Singularities. Degenerating Elliptic Problems		15
0.9. About the Monograph		16
1. Functional Spaces		19
1.1. Notation. Spaces C , S , and S'		19
1.2. Positive and Negative Banach Spaces		22
1.3. Spaces of Bessel Potentials (Liouville Classes) $H^{s,p} = H^{s,p}(\mathbb{R}^n)$		24
1.4. Spaces $H^{s,r,p} = H^{s,r,p}(\mathbb{R}^n)$ ($s, r \in \mathbb{R}$, $1 < p < \infty$)		27
1.5. Besov Spaces $B^{s,p} = B^{s,p}(\mathbb{R}^{n-1})$ ($1 < p < \infty$, $s \in \mathbb{R}$)		29
1.6. Infinitely Smoothing Operators		32
1.7. Spaces $H_{\pm}^{s,r,p} = H_{\pm}^{s,r,p}(\mathbb{R}^n)$ and $H^{s,r,p}(\mathbb{R}_{\pm}^n)$		37
1.8. Spaces $\dot{H}^{s,p}(\mathbb{R}_{\pm}^n)$ ($s \geq 0$, $1 < p < \infty$). Operator Θ^+		41
1.9. Spaces $H^{s,p}(\mathbb{R}_{\pm}^n)$ and $H^{s,r,p}(\mathbb{R}_{\pm}^n)$ and Their Dual Spaces		43
1.10. Space $H^{s,p}(G)$		50
1.11. Space $B^{s,p}(\partial G)$		53
1.12. Operator of Multiplication by a Function in Functional Spaces		55

1.13. Norms that Depend on a Parameter. Spaces $B^s, \ell(G)$ and $W^s, \ell(G)$. Imbedding Theorems	59
1.14. Method of Complex Interpolation. Interpolation Theorem	62
1.15. Noetherian Operators	64
2. Space $\tilde{H}^{s,p,(r)}(\Omega)$	69
2.1. Definition of the Space $\tilde{H}^{s,p,(r)}(\Omega)$	69
2.2. Operator S	70
2.3. Action of Differential Operators upon the Elements of $\tilde{H}^{s,p,(r)}(\Omega)$ (in the Strong Sense)	74
2.4. Action of Differential Operators upon the Elements of $\tilde{H}^{s,p,(r)}(\Omega)$ (in the Weak Sense)	80
2.5. A Modification of Lemma 2.3.1	83
3. Elliptic Boundary-Value Problem	85
3.1. Elliptic Differential Expressions with Constant Coefficients. Proper Ellipticity	85
3.2. Elliptic Boundary-Value Problems in a Half-Space. Lopatinsky Condition	87
3.3. Elliptic Boundary-Value Problem in a Bounded Domain	92
4. Theorem on Complete Collection of Isomorphisms	95
4.1. Principal Theorems	95
4.2. Model Problem in a Half-Space	101
4.3. Solvability of Problem (4.2.26) in $H_+^{s,t,p}(\mathbb{R}^n) = H_+^{s,t,p}$	106
4.4. Proof of Theorem 4.2.1 for $s < 1/p$	110
4.5. Proof of Theorem 4.2.1 for $s \in]1/p, 2m + 1/p[$	120
4.6. Proof of Theorem 4.2.1 for $s > 2m + 1/p$	125
4.7. Smoothing Operator in $\tilde{H}_+^{s,p,(r)}(\mathbb{R}_+^n)$	129
4.8. Regularizer of the Operator $A = (L, B_1, \dots, B_m)$	132
4.9. Completion of the Proof of Theorem 4.1.3	142
5. Elliptic Problems with Normal Boundary Conditions	145
5.1. Normal Systems of Boundary Expressions. Dirichlet Systems	145
5.2. Green's Formula	149
5.3. Regular Elliptic Problems and Adjoint Problems	153
5.4. Strong and Weak Solvability of Regular Elliptic Problems	160
5.5. Regular Elliptic Problems with Homogeneous Boundary Conditions	163

6. Traces of Generalized Solutions of Elliptic Equations on the Boundary of the Domain	173
6.1. Traces of Generalized Solutions of Differential Equations in the Domain with Nowhere Characteristic Boundary	173
6.2. Generalized Solutions Inside a Domain and Their Traces on the Boundary	182
6.3. Traces of Generalized Solutions on Surfaces Parallel to the Boundary	191
7. Local Increase in the Smoothness of Generalized Solutions of Elliptic Boundary-Value Problems. Green's Functions	213
7.1. Increase in the Smoothness of Generalized Solutions in the Entire Domain	213
7.2. Local Increase in Smoothness	215
7.3. Local Increase in the Smoothness of Generalized Solutions of Elliptic Boundary-Value Problems with Homogeneous Boundary Conditions	219
7.4. Green's Function	222
8. Elliptic Problems with Power Singularities on the Right-Hand Sides. Degenerate Elliptic Problems	249
8.1. Regularization of Functions with Point Singularities	250
8.2. Elliptic Problems with Point Power Singularities on the Right-Hand Sides	258
8.3. Regularization of Functions with Power Singularities on i -Dimensional Manifolds ($1 \leq i \leq n - 1$)	267
8.4. Elliptic Problems with Power Singularities on the Right-Hand Sides on i -Dimensional Manifolds	273
8.5. Strongly Degenerate Elliptic Problems on i -Dimensional Manifolds	288
9. Elliptic Boundary-Value Problems with a Parameter	295
9.1. Definition of Elliptic Boundary-Value Problems with a Parameter. Formulation of the Principal Theorem	295
9.2. Model Problem	302
9.3. Proof of Theorem 9.1.1	310
9.4. Applications of Theorem 9.1.1	319
10. Elliptic Boundary-Value Problems for Systems of Equations	323
10.1. Definition of an Elliptic Problem for a System. Principal Theorem	323
10.2. Model Problem in a Half-Space	329
10.3. Proof of Theorem 10.1.1	355
10.4. Traces of Generalized Solutions of Systems of Equations	359
10.5. Local Increase in the Smoothness of Generalized Solutions of Elliptic Systems	378

10.6. Other Applications of the Theorem on Complete Collection of Isomorphisms for Elliptic Systems	381
Bibliographical Notes	385
References	393
Subject Index	413
Notation	415