Contents

Intro	oduction	page 1	
	Part one: Linear Waves	5	
1	Basic Ideas	7	
	Exercises	15	
2	Waves on a Stretched String	17	
2.1	Derivation of the Governing Equation	17	
2.2	Standing Waves on Strings of Finite Length		
2.3	D'Alembert's Solution for Strings of Infinite Length		
2.4	Reflection and Transmission of Waves by Discontinuities in	1	
	Density	29	
	2.4.1 A Single Discontinuity	29	
	2.4.2 Two Discontinuities: Impedance Matching	31	
	Exercises	33	
3	Sound Waves	36	
3.1	Derivation of the Governing Equation	36	
3.2	Plane Waves	40	
3.3	Acoustic Energy Transmission	42	
3.4	Plane Waves In Tubes	45	
3.5	Acoustic Waveguides	50	
	3.5.1 Reflection of a Plane Acoustic Wave by a Rigid Wal	1 50	
	3.5.2 A Planar Waveguide	51	
	3.5.3 A Circular Waveguide	53	
3.6	Acoustic Sources		
	3.6.1 The Acoustic Source	58	
	3.6.2 Energy Radiated by Sources and Plane Waves	62	
3.7	Radiation from Sources in a Plane Wall	64	
	Exercises	70	

vi Contents

4	Linea	Linear Water Waves		
4.1	Deriv	Derivation of the Governing Equations		
4.2	Linea	ar Gravity Waves	78	
	4.2.1	Progressive Gravity Waves	78	
	4.2.2	Standing Gravity Waves	85	
	4.2.3	The Wavemaker	87	
	4.2.4	The Extraction of Energy from Water Waves	91	
4.3	The I	Effect of Surface Tension: Capillary-Gravity Waves	94	
4.4		Waves	97	
4.5	Ship	Waves	99	
4.6	The S	Solution of Initial Value Problems	104	
4.7	Shalle	ow Water Waves: Linear Theory	109	
	4.7.1	The Reflection of Sea Swell by a Step	112	
	4.7.2	Wave Amplification at a Gently Sloping Beach	114	
4.8	Wave	Refraction	117	
	4.8.1	The Kinematics of Slowly Varying Waves	118	
	4.8.2	Wave Refraction at a Gently Sloping Beach	121	
4.9	The I	Effect of Viscosity	123	
	Exerc	rises	124	
5	Wave	s in Elastic Solids	130	
5.1	Deriv	ration of the Governing Equation	130	
5.2	Wave	s in an Infinite Elastic Body	132	
	5.2.1	One-Dimensional Dilatation Waves	133	
	5.2.2	One-Dimensional Rotational Waves	134	
	5.2.3	Plane Waves with General Orientation	134	
5.3	Two-l	Dimensional Waves in Semi-infinite Elastic Bodies	135	
	5.3.1	Normally Loaded Surface	135	
	5.3.2	Stress-Free Surface	137	
5.4	Wave	s in Finite Elastic Bodies	143	
	5.4.1	Flexural Waves in Plates	144	
	5.4.2	Waves in Elastic Rods	148	
	5.4.3	Torsional Waves	150	
	5.4.4	Longitudinal Waves	155	
5.5	The Excitation and Propagation of Elastic Wavefronts*		156	
	5.5.1	Wavefronts Caused by an Internal Line Force in an		
		Unbounded Elastic Body	157	
	5.5.2	Wavefronts Caused by a Point Force on the Free		
		Surface of a Semi-infinite Elastic Body	161	
	Exerci	ISPS	160	

		Contents	vii
6	Elect	romagnetic Waves	173
6.1		ric and Magnetic Forces and Fields	173
6.2	Electrostatics: Gauss's Law		177
6.3	Magnetostatics: Ampère's Law and the Displacement		
	Curre	ent	179
6.4	Elect	romagnetic Induction: Faraday's Law	180
6.5	Plane	Electromagnetic Waves	182
6.6	Cond	uctors and Insulators	186
6.7	Reflection and Transmission at Interfaces		189
	6.7.1	Boundary Conditions at Interfaces	189
	6.7.2	Reflection by a Perfect Conductor	191
	6.7.3	Reflection and Refraction by Insulators	194
6.8	Waveguides		199
	6.8.1	Metal Waveguides	199
	6.8.2	Weakly Guiding Optical Fibres	202
6.9	Radiation		208
	6.9.1	Scalar and Vector Potentials	208
	6.9.2	The Electric Dipole	210
	6.9.3	The Far Field of a Localised Current Distribution	212
	6.9.4	The Centre Fed Linear Antenna	213
	Exerc	ises	216
	Part	two: Nonlinear Waves	219
7	The I	Formation and Propagation of Charle Ways	221

6.4	Electromagnetic Induction: Faraday's Law		
6.5	Plane Electromagnetic Waves		
6.6	Conductors and Insulators		
6.7	Refle	ction and Transmission at Interfaces	189
	6.7.1	Boundary Conditions at Interfaces	189
	6.7.2	Reflection by a Perfect Conductor	191
	6.7.3	Reflection and Refraction by Insulators	194
6.8	Wave	eguides	199
	6.8.1	Metal Waveguides	199
	6.8.2	Weakly Guiding Optical Fibres	202
6.9	Radia	ation	208
	6.9.1	Scalar and Vector Potentials	208
	6.9.2	The Electric Dipole	210
	6.9.3	The Far Field of a Localised Current Distribution	212
	6.9.4	The Centre Fed Linear Antenna	213
	Exerc	cises	216
	Part	two: Nonlinear Waves	219
7	The I	Formation and Propagation of Shock Waves	221
7.1	Traffi	c Waves	221
	7.1.1	Derivation of the Governing Equation	221
	7.1.2	Small Amplitude Disturbances of a Uniform State	224
	7.1.3	The Nonlinear Initial Value Problem	226
	7.1.4	The Speed of the Shock	236
7.2	Comp	pressible Gas Dynamics	239
	7.2.1	Some Essential Thermodynamics	239
	7.2.2	Equations of Motion	243
	7.2.3	Construction of the Characteristic Curves	245
	7.2.4	The Rankine-Hugoniot Relations	249
	7.2.5	Detonations*	256
	Exerc	cises	266
8	Nonli	inear Water Waves	269
8.1	Nonli	inear Shallow Water Waves	269
	8.1.1	The Dam Break Problem	270
	8.1.2	A Shallow Water Bore	275

viii Contents

8.2	The Effect of Nonlinearity on Deep Water Gravity Waves:		
	Stokes' Expansion	280	
8.3	The Korteweg-de Vries Equation for Shallow Water		
	Waves: the Interaction of Nonlinear Steepening and Linear		
	Dispersion	285	
	8.3.1 Derivation of the Korteweg-de Vries Equation	287	
	8.3.2 Travelling Wave Solutions of the KdV Equation	290	
8.4	Nonlinear Capillary Waves	298	
	Exercises	306	
9	Chemical and Electrochemical Waves	308	
9.1	The Law of Mass Action	310	
9.2	Molecular Diffusion	314	
9.3	Reaction–Diffusion Systems	315	
9.4	Autocatalytic Chemical Waves with Unequal Diffusion		
	Coefficients*	326	
	9.4.1 Existence of Travelling Wave Solutions	327	
	9.4.2 Asymptotic Solution for $\delta \ll 1$	330	
9.5	The Transmission of Nerve Impulses: the Fitzhugh-Nagumo		
	Equations	334	
	9.5.1 The Fitzhugh–Nagumo Model	339	
	9.5.2 The Existence of a Threshold	342	
	9.5.3 Travelling Waves	343	
	Exercises	349	
	Part three: Advanced Topics	355	
10	Burgers' Equation: Competition between Wave Steepening		
	and Wave Spreading	357	
10.1	Burgers' Equation for Traffic Flow	357	
10.2	The Effect of Dissipation on Weak Shock Waves in an Ideal		
	Gas	362	
10.3	Simple Solutions of Burgers' Equation	369	
	10.3.1 Travelling Waves	369	
	10.3.2 Asymptotic Solutions for $v \ll 1$	370	
	Exercises	375	
11	Diffraction and Scattering	378	
11.1	Diffraction of Acoustic Waves by a Semi-infinite Barrier	379	
	11.1.1 Preliminary Estimates of the Potential	380	
	11.1.2 Pre-transform Considerations	383	
	11.1.3 The Fourier Transform Solution	385	
11.2	The Diffraction of Waves by an Aperture	391	

Contents	ix
Contents	1X

	11.2.1 Scalar Diffraction: Acoustic Waves	391
	11.2.2 Vector Diffraction: Electromagnetic Waves	394
11.3	Scattering of Linear, Deep Water Waves by a Surface	
	Piercing Cylinder	399
	Exercises	403
12	Solitons and the Inverse Scattering Transform	405
12.1	The Korteweg-de Vries Equation	406
	12.1.1 The Scattering Problem	406
	12.1.2 The Inverse Scattering Problem	410
	12.1.3 Scattering Data for KdV Potentials	416
	12.1.4 Examples: Solutions of the KdV Equation	418
12.2	The Nonlinear Schrödinger Equation	424
	12.2.1 Derivation of the Nonlinear Schrödinger Equation	
	for Plane Electromagnetic Waves	424
	12.2.2 Solitary Wave Solutions of the Nonlinear Schrödinger	
	Equation	431
	12.2.3 The Inverse Scattering Transform for the Nonlinear	
	Schrödinger Equation	435
	Exercises	448
Apper	ndix 1 Useful Mathematical Formulas and Physical Data	451
A1.1	Cartesian Coordinates	451
A1.2	Cylindrical Polar Coordinates	451
A1.3	Spherical Polar Coordinates	452
A1.4	Some Vector Calculus Identities and Useful Results for	
	Smooth Vector Fields	453
A1.5	Physical constants	454
Biblio	ography	455
Indox		459