CONTENTS

PREFACE xi

I. INTRODUCTION 1

§ 1. Review of previous work 1
§ 2. Some theorems on linear equations in Banach spaces 21
§ 3. Stereographic projection 30
§ 4. Completely continuous operators 32

II. SIMPLEST PROPERTIES OF MULTIDIMENSIONAL SINGULAR INTEGRALS 37

§ 5. Basic concepts 37
§ 6. Lipschitz conditions 46
§ 7. Order of singular integrals at infinity 50
§ 8. Differentiation of integrals with a weak singularity 59

III.. COMPOUNDING OF SINGULAR INTEGRALS 63

§ 9. Compounding of singular and ordinary integrals 63
§ 10. Compounding of double singular integrals 67
§ 11. The concept of a singular operator 70
§ 12. Compounding of double singular integrals. The symbol 71
§ 13. Compounding of multidimensional singular integrals 72
§ 14. Formulae for reference 74
§ 15. Product of the operators A_1 and A_n 77
§ 16. Product of the operators A_2 and A_n 81
§ 17. Calculation of $\varkappa_{1,m}$ 83
§ 18. Symbol of a multidimensional singular integral 86

IV. PROPERTIES OF THE SYMBOL 93

§ 19. Fourier transform of a singular kernel 93
§ 20. Fourier transform of a kernel and the symbol of a singular operator 97
§ 21. Transformation of the symbol under change of variables 104

§ 22. Differentiability of the symbol 109
§ 23. The conditions for the continuity of the symbol 112

V. SINGULAR INTEGRALS IN L_p SPACES 116

§ 24. The simplest corollaries from the Fourier transform. First theorem on boundedness in L_2 space 116
§ 25. Symbol dependent on the pole. Second theorem on boundedness in L_2 space 119
§ 26. On the boundedness of a singular integral operator in L_p space 123
§ 27. Integrals taken over any manifold 130
§ 28. Differential properties of singular integrals 131

VI. FURTHER INVESTIGATION OF THE SYMBOL 134

§ 29. More about the differentiation of integrals with a weak singularity 134
§ 30. Polyharmonic potentials 135
§ 31. Series of spherical functions 136
§ 32. Differential properties of the symbol and the characteristic 148
§ 33. Rule for the multiplication of the symbols in the general case 150
§ 34. Conjugate singular operator 154

VII. SINGULAR INTEGRAL EQUATIONS 157

§ 35. The case where the symbol is independent of the pole 157
§ 36. The case where the symbol is dependent on pole. Regularization and domains of constancy of the index 158
§ 37. Equivalent regularization. Index theorem 160
§ 38. Equations with an integral taken over a closed manifold 172
§ 39. Extension by means of the parameter 180
§ 40. Systems of singular integral equations 184
§ 41. Singular integral equations in classes of Lipschitz functions 190

VIII. MISCELLANEOUS APPLICATIONS 199

§ 42. Leading derivatives of volume potential 199
§ 43. Problem of the oblique derivative 203
§ 44. Inequality involving the tangential and normal components of the gradient of a harmonic function 208
§ 45. Equilibrium of an isotropic elastic body 210
§ 46. Diffraction of stationary elastic waves 220

APPENDIX. Multipliers of Fourier integrals 225

BIBLIOGRAPHY 241

INDEX 251

OTHER TITLES IN THE SERIES 257