Contents

Preface

| CHAPTER 1 THE D-OPERATOR | |
|------------------------------------------------------|----|
| 1.1 Operational Techniques | |
| 1.2 Simple D-algebra | 2 |
| 1.3 The Inverse Operator | 4 |
| 1.4 Expansion of the Inverse Operator | (|
| Worked Examples | 7 |
| Exercises | 10 |
| Answers | 11 |
| Chapter 2 The Laplace Integral | 12 |
| 2.1 Conditions for Existence | 12 |
| 2.2 Abscissae of Convergence | 13 |
| 2.3 Elementary Transforms | 14 |
| 2.4 Simple Applications | 17 |
| Worked Examples | 18 |
| Exercises | 20 |
| Answers | 21 |
| CHAPTER 3 SIMPLE PROPERTIES OF THE LAPLACE TRANSFORM | 22 |
| 3.1 Linear Property | 22 |
| 3.2 Change of Scale | 22 |
| 3.3 Shift of the Origin | 23 |
| 3.4 Periodic Functions | 23 |
| 3.5 Multiplication by a Simple Power | 24 |
| 3.6 Expressions involving Inverse Powers of t | 25 |
| 3.7 Transform of Integrals | 25 |
| 3.8 Transform of an Integral Function | 26 |
| 3.9 Examples illustrating 3.8 | 27 |
| 3.10 Behaviour for Large s | 28 |
| Exercises | 28 |
| Answers | 29 |

| iii | CONTENTS |
|-----|----------|

| CHAPTER 4 THE INVERSE INTEGRAL 4.1 The Inversion Problem 4.2 The Fourier Transform 4.3 The Inverse as a Contour Integral 4.4 Examples of Inversion Exercises | 30 30 30 32 35 40 |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------|
| | |
| Chapter 5 The Convolution or Faltung Integral | 43 |
| 5.1 The Repeated Integral | 43 |
| 5.2 Simple Application to Differential Equations | 45 |
| 5.3 A Generalization of 5.2 | 46 |
| 5.4 Transform of a Product | 48 |
| 5.5 A Simple Illustration | 49 |
| Exercises | 49 |
| Answers | 50 |
| Chapter 6 Application to Ordinary Linear Differential | |
| Eouations | 51 |
| 6.1 Limitations of the Technique | 51 |
| 6.2 The Bessel Equation | 52 |
| 6.3 Another Related Equation | 53 |
| 6.4 The Laguerre Equation | 53 |
| 6.5 The Hermite Equation | 57 |
| 6.6 The Laplace Linear Equation | 58 |
| 6.7 A Further Illustrative Example | 60 |
| Exercises | 61 |
| Answers | 62 |
| Chapter 7 Partial Differential Equations | 64 |
| 7.1 Limitations of the Technique | 64 |
| 7.2 A Simple Equation of the First Order | 65 |
| 7.3 The Diffusion Equation | 66 |
| 7.4 The Wave Equation | 68 |
| 7.4.1 The infinite string | 68 |
| 7.4.2 The finite string | 69 |
| 7.4.3 The axially-symmetric equation | 70 |
| 7.4.4 The equation in three dimensions | 71 |
| Exercises | 73 |
| Answers | 74 |

| CONTENTS | ix |
|------------------------------------------------------------------------------------------------------------------------------------|----------|
| CHAPTER 8 LINEAR INTEGRAL EQUATIONS | 75 |
| 8.1 Types of Integral Equations | 75 |
| 8.2 The Difference Kernel | 76 |
| 8.2.1 The equation of the first kind | 76 |
| 8.2.2 The equation of the second kind | 77 |
| 8.3 The Wiener-Hopf Technique | 79 |
| 8.4 The Milne Equation | 82 |
| Exercises | 84 |
| Answers | 85 |
| Chapter 9 Linear Difference Equations | 86 |
| 9.1 The Difference Equations | 86 |
| 9.2 Transform of a Step Function | 87 |
| 9.3 Simple Applications | 87 |
| Worked Examples | 89 |
| 9.4 Inhomogeneous Equations | 90 |
| Worked Examples | 90 |
| 9.5 Variable Coefficients | 91 |
| Worked Examples | 92 |
| Exercises | 93 94 |
| Answers | 94 |
| CHAPTER 10 ASYMPTOTIC FORMULAE | 95 |
| 10.1 Elementary Asymptotic Results | 95 96 |
| 10.2 Asymptotic Form of a Transform for Large s 10.3 Asymptotic Series of a Transform for Large s | 90 97 |
| 10.3 Asymptotic Series of a Transform for Large [5] 10.4 Application to the Exponential Integral | 98 |
| 10.4 Application to the Exponential Integral 10.5 Asymptotic Behaviour of $f(t)$ | 99 |
| 10.5.1 Single-valued function | 99 |
| 10.5.2 Function with a branch point | 100 |
| Worked Examples | 102 |
| 10.6 Terms involving Logarithms | 104 |
| A Simple Example | 104 |
| Exercises | 105 |
| Answers | 106 |

| x | CONTENTS | |
|------------|---------------------------------------------|-----|
| Appendix 1 | THE FOURIER INTEGRAL THEOREM | 107 |
| Appendix 2 | THE EXPONENTIAL INTEGRAL | 112 |
| Appendix 3 | OPERATIONAL FORMULAE | 115 |
| Appendix 4 | Table of the More Common Laplace Transforms | 118 |
| References | | 120 |
| Index | | 121 |