Contents

CHAPTER 1. **Fundamental Concepts**

1.1.	*Introduction*	1
1.2.	*Order Relations*	6
1.3.	*Asymptotic Power Series Expansions*	9
1.4.	*Asymptotic Sequences and Asymptotic Expansions of Poincaré Type*	14
1.5.	*Auxiliary Asymptotic Sequences*	19
1.6.	*Complex Variables and the Stokes Phenomenon*	21
1.7.	*Operations with Asymptotic Expansions of Poincaré Type*	25
1.8.	*Exercises*	33
	References	40

CHAPTER 2. **Asymptotic Expansions of Integrals: Preliminary Discussion**

2.1.	*Introduction*	41
2.2.	*The Gamma and Incomplete Gamma Functions*	42
2.3.	*Integrals Arising in Probability Theory*	44
2.4.	*Laplace Transform*	46
2.5.	*Generalized Laplace Transform*	50
2.6.	*Wave Propagation in Dispersive Media*	53
2.7.	*The Kirchhoff Method in Acoustical Scattering*	55
2.8.	*Fourier Series*	60
2.9.	*Exercises*	62
	References	68

CHAPTER 3. **Integration by Parts**

3.1.	*General Results*	69
3.2.	*A Class of Integral Transforms*	76
3.3.	*Identification and Isolation of Critical Points*	84
3.4.	*An Extension of the Integration by Parts Procedure*	89
3.5.	*Exercises*	92
	References	101

CHAPTER 4. *h*-Transforms with Kernels of Monotonic Argument

4.1. Laplace Transforms and Watson's Lemma 102
4.2. Results on Mellin Transforms 106
4.3. Analytic Continuation of Mellin Transforms 110
4.4. Asymptotic Expansions for Real λ 117
4.5. Asymptotic Expansions for Real λ : Continuation 125
4.6. Asymptotic Expansions for Small Real λ 130
4.7. Asymptotic Expansions for Complex λ 136
4.8. Electrostatics 146
4.9. Heat Conduction in a Nonlinearly Radiating Solid 152
4.10. Fractional Integrals and Integral Equations of Abel Type 155
4.11. Renewal Processes 159
4.12. Exercises 165
References 178

CHAPTER 5. *h*-Transforms with Kernels of Nonmonotonic Argument

5.1. Laplace's Method 180
5.2. Kernels of Exponential Type 187
5.3. Kernels of Exponential Type: Continuation 195
5.4. Kernels of Algebraic Type 199
5.5. Expansions for Small λ 206
5.6. Exercises 209
References 218

CHAPTER 6. *h*-Transforms with Oscillatory Kernels

6.1. Fourier Integrals and the Method of Stationary Phase 219
6.2. Further Results on Mellin Transforms 224
6.3. Kernels of Oscillatory Type 229
6.4. Oscillatory Kernels: Continuation 238
6.5. Exercises 245
References 251

CHAPTER 7. The Method of Steepest Descents

7.1. Preliminary Results 252
7.2. The Method of Steepest Descents 262
7.3. The Airy Function for Complex Argument 281
7.4. The Gamma Function for Complex Argument 286
7.5. The Klein-Gordon Equation 292
7.6. The Central Limit Theorem for Identically Distributed Random Variables 298

7.7. *Exercises* 302
References 319

CHAPTER 8. **Asymptotic Expansions of Multiple Integrals**

8.1. *Introduction* 321
8.2. *Asymptotic Expansion of Double Integrals of Laplace Type* 322
8.3. *Higher-Dimensional Integrals of Laplace Type* 331
8.4. *Multiple Integrals of Fourier Type* 340
8.5. *Parametric Expansions* 354
8.6. *Exercises* 359
References 366

CHAPTER 9. **Uniform Asymptotic Expansions**

9.1. *Introduction* 367
9.2. *Asymptotic Expansions of Integrals with Two Nearby Saddle Points* 369
9.3. *Underlying Principles* 379
9.4. *Saddle Point near an Amplitude Critical Point* 380
9.5. *A Class of Integrals That Arise in the Analysis of Precursors* 387
9.6. *Double Integrals of Fourier Type* 393
9.7. *Exercises* 401
References 411

Appendix 413
General References 417
Index 421