

Contents

1	Introduction	1
1.1	Overview	2
1.2	Outline of this Monograph	3
1.3	Background References	5
2	The Entropy of a System	7
2.1	Introduction	8
2.2	Definition of the Entropy	8
2.3	Some Properties	9
2.4	Relations to the LQG Cost	11
2.5	The \mathcal{H}_∞ -Norm Bound	12
3	The Minimum Entropy \mathcal{H}_∞ Control Problem	15
3.1	Introduction	16
3.2	Problem Formulation	16
3.3	Solution in the General Case	19
3.4	Solution at Infinity	22
3.5	State-Space Formulae for the Minimum Entropy	25
3.6	Upper Bounds and an \mathcal{H}_∞ /LQG Tradeoff	30
3.7	Recovery of the LQG Solution	31
4	The Minimum Entropy \mathcal{H}_∞ Distance Problem	35
4.1	Introduction	36
4.2	From \mathcal{H}_∞ Control Problem to \mathcal{H}_∞ Distance Problem	36
4.3	Relations to the Band Extension Problem	38
4.4	Solution in the General Case	38
4.5	Solution at Infinity	42
5	Relations to Combined \mathcal{H}_∞/LQG Control	49
5.1	Introduction	50
5.2	The Combined \mathcal{H}_∞ /LQG Problem	50
5.3	Equivalence with Minimum Entropy \mathcal{H}_∞ Control	52
5.4	Solution and Equivalence in State-Space	54

6	Relations to Risk-Sensitive LQG Control	59
6.1	Introduction	60
6.2	The Risk-Sensitive LQG Control Problem	60
6.3	Equivalence with Minimum Entropy \mathcal{H}_∞ Control	61
7	The Normalized \mathcal{H}_∞ Control Problem	65
7.1	Introduction	66
7.2	The Normalized LQG Problem	66
7.3	The Normalized \mathcal{H}_∞ Problem	68
7.4	A Numerical Example	72
8	\mathcal{H}_∞-Characteristic Values	79
8.1	Introduction	80
8.2	LQG-Balancing and LQG-Characteristic Values	81
8.3	\mathcal{H}_∞ -Balancing and \mathcal{H}_∞ -Characteristic Values	83
8.4	Model Reduction by \mathcal{H}_∞ -Balanced Truncation	86
8.4.1	\mathcal{H}_∞ -Balanced Truncation	86
8.4.2	Relations to Balanced Truncation	87
8.4.3	Coprime Factorization via the Normalized \mathcal{H}_∞ Problem	90
8.4.4	Model Reduction via the Coprime Factors	93
8.4.5	Stability and Performance with the Reduced-Order Controller	95
8.4.6	A Numerical Example	99
9	LQG and \mathcal{H}_∞ Monotonicity	103
9.1	Introduction	104
9.2	The LQG Cost and its Derivative	104
9.3	On LQG Monotonicity	108
A	Proof of Results Needed in the Text	111
A.1	Outline	112
A.2	A Lemma	112
A.3	Proof of Theorem 2.4.4	112
A.4	State-Space Evaluation of the Entropy Integral	113
A.5	Proof of Lemma 8.4.9	116
A.6	Proof of Corollary 8.4.19 and Proposition 8.4.21	118
A.7	Proof of Theorem 9.2.2(ii)	121
B	Entropy Formulae: Alternative Derivation	123
B.1	Introduction	124
B.2	The Full Information and Output Estimation Problems	124
B.3	Separation Structure	126
B.4	Proof of Theorem 3.5.1	127
B.5	Proof of Lemma 3.5.5	128

C Notation	131
C.1 Basic Notational Conventions	132
C.2 Acronyms	134
Bibliography	135
Index	141